

# Calibration of an expeditious terramechanics model using a higher-fidelity model, Bayesian inference, and a virtual bevameter test

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## EXTENDED ABSTRACT

### 1 Introduction

The soil contact model (SCM) is widely used in practice for off-road wheeled vehicle mobility studies when simulation speed is important and highly accurate results are not a main concern. In practice, the SCM parameters are obtained via a bevameter test, which requires a complex apparatus and experimental procedure. Here, we advance the idea of running a virtual bevameter test using a high-fidelity terramechanics simulation. The latter employs the “continuous representation model” (CRM), which regards the deformable terrain as an elasto-plastic continuum that is spatially discretized using the smoothed particle hydrodynamics (SPH) method. The approach embraced is as follows: a virtual bevameter test is run in simulation using CRM terrain to generate “ground truth” data; in a Bayesian framework, this data is subsequently used to calibrate the SCM terrain. We show that (i) the resulting SCM terrain, while leading to fast terramechanics simulations, serves as a good proxy for the more complex CRM terrain; and (ii) the SCM-over-CRM simulation speedup is roughly one order of magnitude. These conclusions are reached in conjunction with two tests: a single wheel test, and a full VIPER rover simulation. The SCM and CRM simulations are run in an open-source software called Chrono.

### 2 Numerical approach

The proposed calibration methodology, which uses a virtual bevameter test, relies on two terramechanics models – SCM and CRM, and draws on a Bayesian calibration framework implemented in a software package called PyMC. The SCM approach embraced draws on the Bekker-Wong formula, which relates the normal pressure  $p$  to the sinkage  $z$  for a wheel of width  $b$  using a semi-empirical, experiment-based curve fitting with parameters  $K_c$ ,  $K_\phi$ , and  $n$ :

$$p = \left( \frac{K_c}{b} + K_\phi \right) z^n. \quad (1)$$

The Bekker-Wong formula is combined with the Janosi-Hanamoto formula, the latter used to evaluate the shear stress between the wheel and terrain:

$$\tau = (c + p \tan \varphi) (1 - e^{-J_s/K_s}), \quad (2)$$

where  $J_s$  is the accumulated shear displacement,  $c$  is the cohesion coefficient,  $\varphi$  the internal friction angle, and  $K_s$  the so-called Janosi parameter. For CRM, we employ a homogenization of the granular material and use an elasto-plastic continuum model to capture the dynamics of the deformable terrain. Herein, the CRM solution is obtained using the SPH method, which is a Lagrangian particle-based solution that requires no background grid. The SPH method has proven effective and efficient in simulating granular material problems with large deformation. In CRM, the problem unknowns, i.e., field velocity vector  $\mathbf{u}$  and the Cauchy stress tensor  $\boldsymbol{\sigma}$ , enter the mass and momentum balance equations as:

$$\begin{cases} \frac{d\mathbf{u}}{dt} = \frac{\nabla \boldsymbol{\sigma}}{\rho} + \mathbf{f}_b \\ \frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{u} \end{cases}, \quad (3)$$

where  $\rho$  is the density of the deformable terrain, and  $\mathbf{f}_b$  represents external forces, e.g., the gravity force. The total stress tensor  $\boldsymbol{\sigma} \in \mathbb{R}^{3 \times 3}$  is split in two components expressed as  $\boldsymbol{\sigma} \equiv -p\mathbf{I} + \boldsymbol{\tau}$ , where  $\boldsymbol{\tau}$  is the deviatoric component of the total stress tensor and  $p$  is the pressure which can be calculated from the trace of the total stress tensor as  $p = -\frac{1}{3} \text{tr}(\boldsymbol{\sigma}) = -\frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$ . For closure, a stress rate tensor formula is employed, which is expressed as:

$$\frac{d\boldsymbol{\sigma}}{dt} = \dot{\boldsymbol{\phi}} \cdot \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\phi}} + 2G[\dot{\boldsymbol{\epsilon}} - \frac{1}{3} \text{tr}(\dot{\boldsymbol{\epsilon}})\mathbf{I}] + \frac{1}{3} K \text{tr}(\dot{\boldsymbol{\epsilon}})\mathbf{I}. \quad (4)$$

### 3 Calibration of the SCM model using a virtual bevameter

While the SPH-backed CRM approach posts real-time factor (RTF) values of 30 and above, the SCM implementation in Chrono can achieve real-time or close (RTF is defined as the amount of time needed to simulate a second of the dynamics of a system). Moreover, the SCM approach in many cases can adequately capture the wheel/soil interaction. However, the parameters tied to the SCM model are usually unknown and need to be first calibrated via a bevameter test. The actual bevameter is

a self contained unit designed to take in-situ soil strength measurements, usually in conjunction with vehicle mobility modeling and simulation. The bevameter consists of two devices – a plate sinkage device, and an annulus shear device. Since the CRM parameters are physics based, e.g., Young’s modulus, friction angle, we use these parameters to carry out a virtual bevameter test that generates the “experimental” data. Using data obtained with the virtual bevameter, we estimated the parameters of the SCM model in a two-step approach. First, data obtained using a virtual bevameter sinkage test is used to calibrate  $K_c$ ,  $K_\phi$ , and  $n$  in Eq. (1); these parameters affect the wheel-soil normal contact force. Subsequently, we calibrate the other three parameters ( $c$ ,  $\phi$ , and  $K_s$  in Eq. (2)) for the tangential force evaluation using data obtained from the annulus shear test. Table 1 reports values for all six SCM parameters calibrated via Bayesian inference using data generated by a virtual bevameter test with CRM terrain.

Table 1: Calibrated values of the SCM parameters using data generated from CRM simulations.

SCM parameter	$K_c$ (N/m <sup>n+1</sup> )	$K_\phi$ (N/m <sup>n+2</sup> )	$n$	$c$ (Pa)	$\phi$ (deg)	$K_s$ (m)
Calibrated value	-1.1e5	2.2e6	1.2	2495	24	2.95e-3

#### 4 Validation of the calibrated SCM model

The SCM parameters obtained using the virtual bevameter test are used in this section to conduct single wheel and full VIPER rover simulations. The SCM and CRM simulation results compared are the DrawBar-Pull force and terrain slope that the wheel/rover is able to negotiate. The single wheel and full VIPER rover were moved under a controlled slip and normal loading conditions within a confined soil bin, see Fig. 1 (a) and (b). The images also shows the simulation results obtained using the CRM approach with wheel/rover position and the particle distributions of the terrain. The SCM and CRM DrawBar-Pull force are shown in Fig. 1 (c) and (d), which indicates a good agreement. More results can be found in [3].

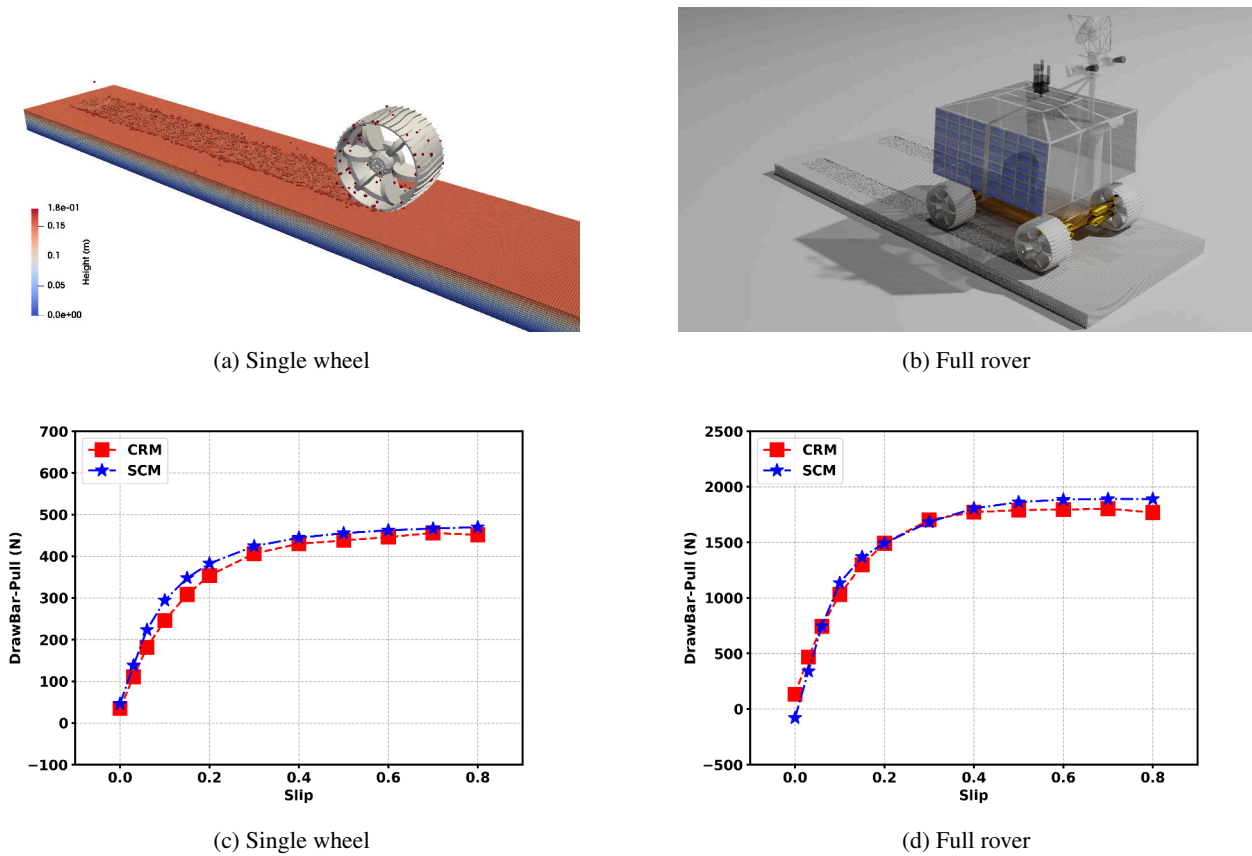


Figure 1: Screenshots, DrawBar-Pull force vs. slip of single wheel and full rover simulations.

#### References

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- [3] Dan Negrut, Wei Hu, Pei Li, et al. Calibration of an expeditious terramechanics model using a higher-fidelity model, Bayesian inference, and a virtual bevameter test. *Authorea*. January 12, 2023.