A three-sub-step composite method for the dynamic analysis of rigid-flexible multibody systems

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EXTENDED ABSTRACT

1 Introduction

Due to the complexity of practical problems, analytical solutions are generally not available for transient analyses, thus numerical methods are pre-dominantly used to approximate the transient response. Time integration methods [1] are a widely-used numerical tool for the dynamics analysis of transient problems, such as structural dynamics and multibody dynamics. In the research of multibody systems, the widely-used time integration methods are the Newmark method, Generalized-α method, HHT-α method, Runge-Kutta method, and backward difference formula (BDF) in the past decades. To our knowledge, there are many excellent methods for structural dynamics, including the parameter methods, energy conserving methods, multistep methods, and composite methods. Using them to simulate multibody systems seems natural. Among these methods, the self-starting composite methods possessing advantages in accuracy, efficiency, dissipation, and stability have become more attractive in recent decades. The three-sub-step composite method TTBIF [2] proposed by the present authors has been proved to have excellent performance in linear and nonlinear structural dynamics.

In this context, the three-sub-step TTBIF is further optimized and generalized to rigid-flexible multibody systems wherein rigid body rotations are described by Euler parameters in this work. For exactly computing the angular velocity that does not exist in structural dynamics, the stepping equations of the TTBIF are modified in terms of the relation between Euler parameters and angular velocity. In addition, the new structure of the algorithms can eliminate numerical drifts at the levels of displacement and velocity. Theoretical analysis finds that the newly developed TTBIF for rigid flexible multibody systems has second-order and controllable numerical dissipation. Compared to the time integration methods that have been widely used in the simulations of rigid-flexible multibody systems, the newly developed TTBIF enjoys the advantages in stability, accuracy, dissipation, efficiency and energy conservation.

2 Governing equations

The governing equations of multibody systems are generally described as

$$\boldsymbol{M}\left(\boldsymbol{q}\right)\ddot{\boldsymbol{q}}+\boldsymbol{C}_{\boldsymbol{q}}^{\mathrm{T}}\left(\boldsymbol{q}\right)\boldsymbol{\lambda}=\boldsymbol{Q}\left(\dot{\boldsymbol{q}},\boldsymbol{q},t\right),\quad \boldsymbol{C}\left(\boldsymbol{q},t\right)=\boldsymbol{0} \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the position constraint equations; $\boldsymbol{C}_{\boldsymbol{q}}(\boldsymbol{q}, t)= \partial\boldsymbol{C}(\boldsymbol{q}, t)/\partial\boldsymbol{q}$ is the constraint's Jacobian matrix; $\boldsymbol{Q}$ is the generalized force vector; $\boldsymbol{\lambda}$ is the Lagrange multiplier vector; the generalized coordinate vector $\boldsymbol{q}$ contains translational, rotational and flexible components for every body of the system. In this work, the ANCF (Absolute Nodal Coordinate Formulation) elements are adopted for space discretization. The coordinates of ANCF element nodes are defined in the global coordinate frame, and the angle coordinates of nodes are described by the slope vector. Hence, motion equations discretized by ANCF elements can be directly solved by time integration methods developed for structural dynamics. The components of $\boldsymbol{q}$ with ANCF elements can be directly discretized by the TTBIF, and time-stepping formulations have the forms as

$$\boldsymbol{q}_{t+\gamma\Delta t}=\boldsymbol{q}_t+\gamma\Delta t\dot{\boldsymbol{q}}_t+\frac{1}{4}\gamma^2\Delta t^2\left(\ddot{\boldsymbol{q}}_t+\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}\right),\quad \dot{\boldsymbol{q}}_{t+\gamma\Delta t}=\dot{\boldsymbol{q}}_t+\frac{1}{2}\gamma\Delta t\left(\ddot{\boldsymbol{q}}_t+\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}\right) \tag{2}$$

$$\boldsymbol{q}_{t+2\gamma\Delta t}=\boldsymbol{q}_{t+\gamma\Delta t}+\gamma\Delta t\dot{\boldsymbol{q}}_{t+\gamma\Delta t}+\frac{1}{4}\gamma^2\Delta t^2\left(\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}+\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t}\right),\quad \dot{\boldsymbol{q}}_{t+2\gamma\Delta t}=\dot{\boldsymbol{q}}_{t+\gamma\Delta t}+\frac{1}{2}\gamma\Delta t\left(\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}+\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t}\right) \tag{3}$$

$$\begin{aligned}\boldsymbol{q}_{t+\Delta t}&=\boldsymbol{q}_t+\Delta t\left[\left(\theta_0+\theta_3\right)\dot{\boldsymbol{q}}_t+\theta_1\dot{\boldsymbol{q}}_{t+\gamma\Delta t}+\theta_2\dot{\boldsymbol{q}}_{t+2\gamma\Delta t}+\theta_3\Delta t\left(\theta_0\ddot{\boldsymbol{q}}_t+\theta_1\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}+\theta_2\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t}+\theta_3\ddot{\boldsymbol{q}}_{t+\Delta t}\right)\right],\\ \dot{\boldsymbol{q}}_{t+\Delta t}&=\dot{\boldsymbol{q}}_t+\Delta t\left(\theta_0\ddot{\boldsymbol{q}}_t+\theta_1\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}+\theta_2\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t}+\theta_3\ddot{\boldsymbol{q}}_{t+\Delta t}\right)\end{aligned} \tag{4}$$

For exactly describing the angular velocity, the expressions of velocity are modified as applied to the calculations of the components of $\boldsymbol{q}$ with the Euler parameters. The formulations of the modified TTBIF (noted as mTTBIF) for rigid body rotations have the forms as

$$\dot{\boldsymbol{q}}_{t+\gamma\Delta t}=\boldsymbol{L}_{t+\gamma\Delta t}^{\mathrm{T}}\left(\boldsymbol{L}_t\dot{\boldsymbol{q}}_t+0.5\gamma\Delta t\left(\boldsymbol{L}_t\ddot{\boldsymbol{q}}_t+\boldsymbol{L}_{t+\gamma\Delta t}\ddot{\boldsymbol{q}}_{t+\gamma\Delta t}\right)\right) \tag{5}$$

$$\dot{\boldsymbol{q}}_{t+2\gamma\Delta t} = \boldsymbol{L}^{\mathrm{T}}_{t+2\gamma\Delta t}\left(\boldsymbol{L}_{t+\gamma\Delta t}\dot{\boldsymbol{q}}_{t+\gamma\Delta t} + 0.5\gamma\Delta t\left(\boldsymbol{L}_{t+\gamma\Delta t}\ddot{\boldsymbol{q}}_{t+\gamma\Delta t} + \boldsymbol{L}_{t+2\gamma\Delta t}\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t}\right)\right) \tag{6}$$

$$\dot{\boldsymbol{q}}_{t+\Delta t} = \boldsymbol{L}^{\mathrm{T}}_{t+\Delta t}\left(\boldsymbol{L}_t\dot{\boldsymbol{q}}_t + \Delta t\left(\theta_0\boldsymbol{L}_t\ddot{\boldsymbol{q}}_t + \theta_1\boldsymbol{L}_{t+\gamma\Delta t}\ddot{\boldsymbol{q}}_{t+\gamma\Delta t} + \theta_2\boldsymbol{L}_{t+2\gamma\Delta t}\ddot{\boldsymbol{q}}_{t+2\gamma\Delta t} + \theta_3\boldsymbol{L}_{t+\Delta t}\ddot{\boldsymbol{q}}_{t+\Delta t}\right)\right) \tag{7}$$

3 Numerical experiment

The example deals with the flexible multibody model of a satellite, as shown in Fig. 1. The satellite is comprised of a rigid body as well as nonlinear beam components, and the rigid body and the flexible components are connected by spherical hinges. Starting at reset, an external moment M=3000N·m is applied to the central rigid body. After t=0.01s, no external moment act on the system anymore such that the total angular momentum is a conserved quantity. The performances of the Generalized-α (ρ_∞=0.8), the combination of the HHT-α and iHHT-α (α=-0.1), and the combination of the TTBIF and mTTBIF (ρ_∞=0.8) are compared. The displacements of these methods are shown in Fig. 2, and we can see that the accuracy of the combination of the TTBIF and mTTBIF is the highest. Also the CPUs of these methods are: CPU(Generalized-α)= 6.4150e+03, CPU(HHT-α and iHHT-α)= 2.6378e+03, CPU(TTBIF and mTTBIF)= 1.6711 e+03, one can find that the computation costs of the combination of the TTBIF and mTTBIF is the lowest.

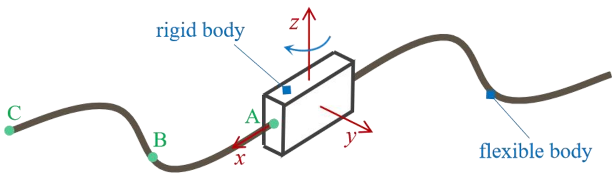

Figure 1: The flexible multibody model of a satellite

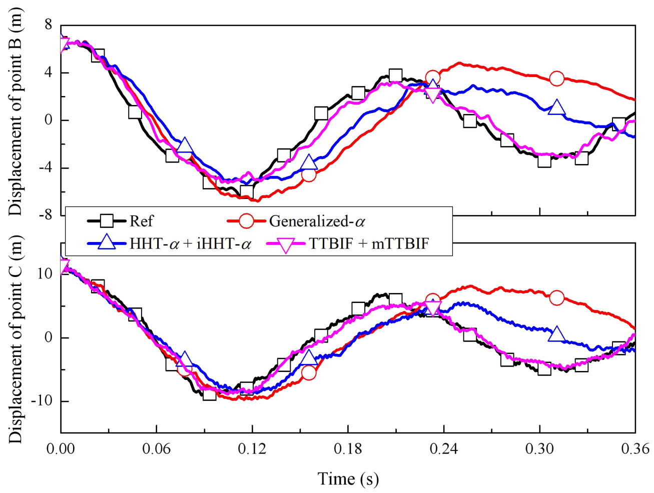

Figure 2: Displacements of the Generalized-α method, the (HHT-α+iHHT-α)method, and the (TTBIF+mTTBIF)

4 Conclusions

This work developed a self-starting three-sub-step composite method for the dynamic analysis of rigid flexible multibody systems. The proposed method achieves second-order accuracy and controllable dissipation, and the motion equations at time discrete points are strictly satisfied. Additionally, the numerical drifts at the displacement and velocity levels can be completely eliminated. The numerical experiments demonstrated that compared to the widely-used Generalized-α method and HHT-α method, the newly constructed method enjoys advantages in accuracy, dissipation, stability, efficiency, and energy-conservation.

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