Employing a Variable Modal Basis Using the Flexible Natural Coordinates Formulation

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EXTENDED ABSTRACT

1 Introduction

Small deformation flexible multibody simulations are employed in many engineering problems, for example in automotive simulations. Typically, the descriptions for such simulations revert to the use of a modal basis for the model order reduction e.g. floating frame of reference (FFR), generalized coordinate mode synthesis (GCMS) or flexible natural coordinates formulation (FNCF) [1]. However, the selection of the modal basis classically poses a challenge, as the relevant modeshapes are typically not known beforehand. Furthermore, the relevancy of the modeshapes mainly depends on the environmental conditions, which may vary throughout the simulation. In many cases, the user either carries out an experimental modal analysis ahead of time, or takes a large set of modeshapes into account. Either way, a lot of effort is required to determine the preferred modal basis.

In order to reduce the required effort and to further optimize the selected set of modeshapes, we propose the use of a variable modal basis. This approach allows a minimization of the number of modeshapes at every instance during the simulation, while still maintaining a predetermined level of accuracy. The proposed framework distinguishes the relevant modeshapes based on their individual energy content. All irrelevant modeshapes are then neglected from the modal basis by reducing the size of the matrices and vectors corresponding to the constraint and dynamic equations. These smaller matrices and vectors allow a reduction in computational load, effectively speeding up the simulation. Modeshapes that are neglected during the simulation have no energy associated to them, however, it is still possible for modeshapes to become reactivated at a later stage in the simulation. If the total energy, associated with the modeshapes rises at any point during the simulation, all modeshapes become active for one timestep, allowing the reduction process to start again. This framework leads to an efficient approach which is validated numerically on a rotor-drum example.

2 Flexible natural coordinates formulation with variable modal basis

The flexible natural coordinates formulation regarded in this work contains a redundant kinematic description with both local $\mathbf{R}_\delta \in \mathbb{R}^{n\times m}$ and global $\mathbf{R}_\gamma \in \mathbb{R}^{n\times 9m}$ modeshapes. The corresponding set of equations of motion has the form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \frac{\partial \mathbf{c}^{\mathbf{T}}}{\partial q}\lambda = \mathbf{f}(\mathbf{q}) \tag{1}$$

$$\mathbf{c}(\mathbf{q}) = 0, \tag{2}$$

where $\mathbf{q}$ consists of translational amplitudes $\mathbf{q_t} \in \mathbb{R}^3$, rotational amplitudes $\mathbf{q_r} \in \mathbb{R}^9$, local participation factors $\mathbf{q}_\delta \in \mathbb{R}^m$ and redundant global participation factors $\mathbf{q}_\gamma \in \mathbb{R}^{9m}$. The energy associated with each modeshape in the modal basis is determined as a combination of kinetic and elastic energy:

$$\mathbf{E_f} = \frac{1}{2}\begin{bmatrix} \dot{\mathbf{q}}_\delta \\ \dot{\mathbf{q}}_\gamma \end{bmatrix} \cdot \mathbf{M_f} \begin{bmatrix} \dot{\mathbf{q}}_\delta \\ \dot{\mathbf{q}}_\gamma \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \mathbf{q}_\delta \\ \mathbf{q}_\gamma \end{bmatrix} \cdot \mathbf{K_f} \begin{bmatrix} \mathbf{q}_\delta \\ \mathbf{q}_\gamma \end{bmatrix} \tag{3}$$

with $\mathbf{M_f} \in \mathbb{R}^{10m\times 10m}$ and $\mathbf{K_f} \in \mathbb{R}^{10m\times 10m}$ the section of the generalized mass and stiffness matrix associated with the flexible degrees of freedom and $\mathbf{E_f} \in \mathbb{R}^{10m}$ containing 10 energy contributions for every modeshape. With this energy distribution, the most relevant modeshapes are selected based on an energy threshold. This threshold is either an absolute minimal energy or a relative threshold compared to the total flexible energy. Any modeshape with less energy than the proposed threshold will become inactive. Hence, the initial modal basis reduces to a smaller set of active modeshapes $\mathbf{R}'_\delta \in \mathbb{R}^{n\times m'}$ and $\mathbf{R}'_\gamma \in \mathbb{R}^{n\times 9m'}$, with corresponding participation factors $\mathbf{q}'_\delta \in \mathbb{R}^{m'}$ and $\mathbf{q}'_\gamma \in \mathbb{R}^{9m'}$. This means that the equations of motion in Eq. (1) and (2) undergo a parallel reduction:

$$\mathbf{M}'\ddot{\mathbf{q}}' + \mathbf{K}'\mathbf{q}' - \frac{\partial \mathbf{c}'^{\mathbf{T}}}{\partial q}\lambda' = \mathbf{f}'(\mathbf{q}') \tag{4}$$

$$\mathbf{c}'(\mathbf{q}') = 0, \tag{5}$$

with $\mathbf{M}', \mathbf{K}' \in \mathbb{R}^{(12+10m')\times(12+10m')}$.

3 Numerical validation

For the numerical validation we consider a rotor inside a drum, loaded with spheres, as shown in Figure 1. Under operational conditions, the rotor has no initial rotational velocity. After $0.2s$ the rotor speeds up until $0.3s$, when the desired rotational velocity of $100rpm$ is reached. As the chaotic movement of the spheres provides a changing environment, various contact loads are applied. For comparison purposes, the loading conditions are simplified to cyclic forces. The rotor is represented by a 2D triangular mesh with 1560 elements and 848 nodes. The flexible multibody model contains an initial modal basis consisting of fifty free-free modeshapes for the rotor.

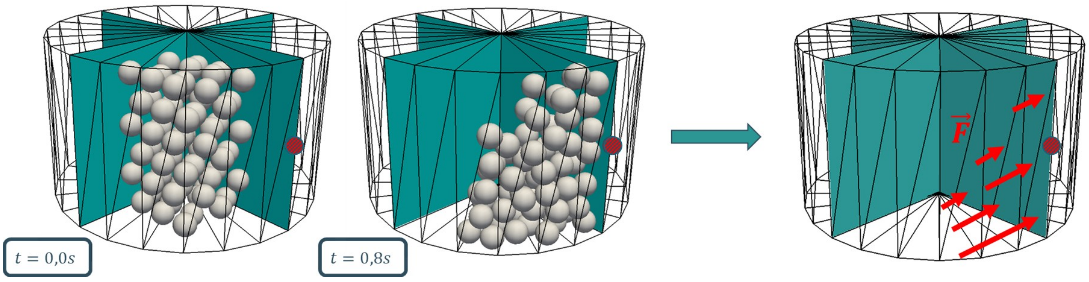

Figure 1: Left: Rotor-drum with 72 spheres at $0.0s$ and $0.8s$. Right: Simplified rotor-drum loading conditions. Node 430 is indicated with a dot.

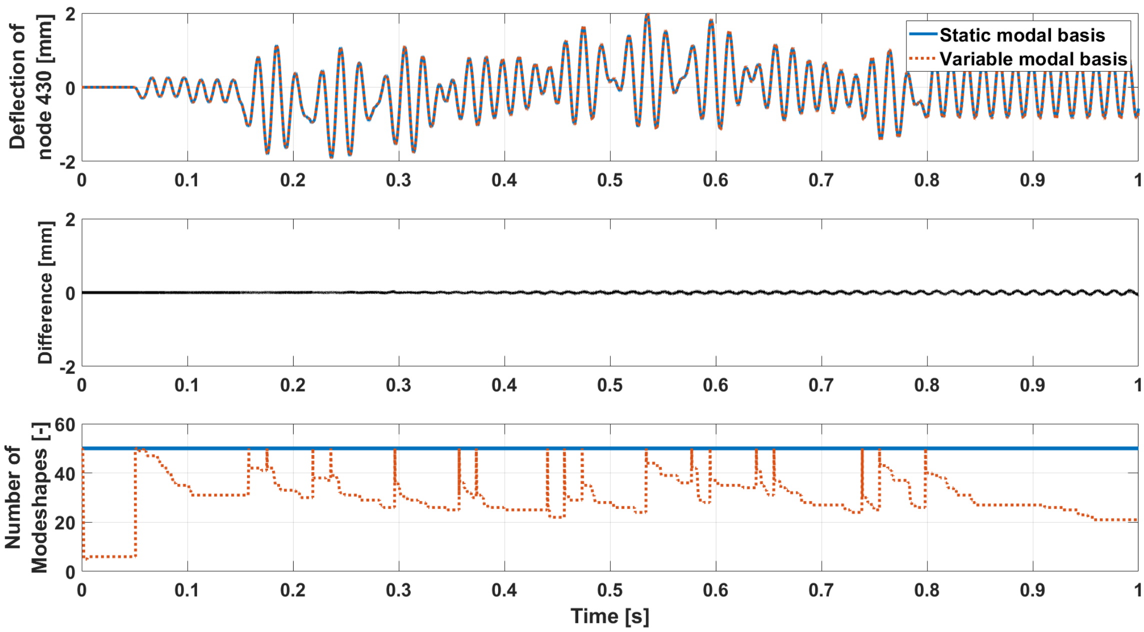

Figure 2: Deflection of the rotor with corresponding active modeshapes in time for both a static and variable modal basis.

The simulation is performed both with a fixed and variable modal basis. Figure 2 compares the deformation located at node 430 on the rotor between both simulations and provides the variation of the number of active modeshapes in time. As a result, the variable modal basis uses the minimum required set of modeshapes to still ensure an accurate description.

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References

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