Self-excitability of roller coaster trains along spatial trajectories

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EXTENDED ABSTRACT

1 Introduction

It is well known that vibrations usually appear during a roller coaster ride, which in some cases leads to passenger discomfort and poses additional material and structural strain. However, the causes of such vibrations are still not fully understood. While there have been technological advances that help to mitigate them to some degree, for instance by placing shock absorbers between the wheels and the bogie frame, an analysis of this phenomenon is required to make better decisions at the design stage.

A simple extension to a perfectly rigid formulation is the lumped mass approach, as proposed in [1] for a quarter-carbody model and in [2] for a two-wheel car. Also, a complete roller coaster train of rigid bodies with a compliant contact model was studied in [3], using real rail trajectory deviation data. Nonetheless, a mismatch between the simulated spatial frequencies and the measurements was observed.

For the first time to the knowledge of the authors, in this investigation the problem is analyzed from the point of view of a self-excited dynamical system, thus ruling out trajectory deviations or local defects. A mechanical model is proposed, consisting in the separation of the overall spatial trajectory motion from the relative motions between train components and the representation of the wrenches at the end bodies of the kinematical chain using a simplified bogie-rail contact approach. Furthermore, the contact problem is extended with respect to previous works by considering the creep forces in the contact interface.

2 Problem formulation

Let n be the number of cars of a roller coaster train, $\underline{r}_{CL}(s) \in \mathbb{R}^3$ the arc-length parametrized spatial curve describing the track centerline and $\underline{\mathrm{R}}_{CL}(s) = \left[\begin{array}{ccc} \hat{\underline{t}} & \hat{\underline{b}} & \hat{\underline{n}} \end{array}\right] \in SO(3)$ the rotation matrix framing the curve, where $\mathrm{d}\underline{r}_{CL}(s)/\mathrm{d}s = \hat{\underline{t}}(s)$. Also let $\underline{q} \in \mathbb{R}^{f(n)}$ be the vector of generalized coordinates describing the configuration of the system, which depends on the specific roller coaster train. The ideal system is described as the single degree of freedom mechanism uniquely determined by the coordinate s_Z that locates one of its bodies (for example, the front axle) with respect to the centerline, so that $\underline{q} = \left[\begin{array}{cc} s_Z & \underline{q}_d \end{array}\right]$. The generalized coordinates $\underline{q}_d$ can be solved for a fixed s_Z using the augmented constraint equations $\underline{\Phi}(\underline{q}_d, \underline{s}_C; s_Z) = \underline{0}$, where $\underline{s}_C$ is a vector of centerline coordinates referred to the bodies that close the kinematical chain. This results in the mappings $q_i^{(0)} = q_i^{(0)}(s_Z)$, for $i \in \{2, \ldots, f(n)\}$, where the superscript (0) denotes the ideal solution. These can be used to initialize the open kinematical chain system (here termed real system) at arbitrary track locations. Furthermore, the dynamics of the ideal system can be solved to obtain a reference trajectory $s_Z(t)$, which may be used in the real system to move the train along the track. This makes it possible to compare different scenarios at the same track locations and speeds.

The real system consists of an open kinematical chain, which may be described by the same generalized coordinates $\underline{q}$ as the ideal system or not, depending on the adopted simplifications. Here, it is assumed that both systems are described by the same $\underline{q}$ and that the end bodies are the wheels. The closest rail point to a reference point in the bogie can be tracked, from which the contact point of each one of the 6 wheels is approximated. Then, the contact normal direction is computed, assuming a cylinder-cylinder contact type, and the contact speed is decomposed in a local reference frame, with the wheel heading direction as the forward axis. This defines the contact penetration and the creepages, yielding the contact forces after applying a proper formulation. In this study, a volume-based reaction is used for the normal contact, and Polach's method for the creep forces.

The equations of motion take the form $\underline{\mathrm{M}}(\underline{q})\dot{\underline{u}} + \underline{f}(\underline{u}, \underline{q}) = \underline{0}$, where $\underline{u}$ is the vector of generalized speeds, $\underline{M}$ is the generalized mass matrix and $\underline{f}$ is the forcing term including the Coriolis and centrifugal terms and the external generalized forces. The parent$(i-1)$-child(i) kinematical relations of the underlying system can be expressed in the form $\underline{\mathrm{R}}_i = \underline{\mathrm{R}}_i(\underline{\mathrm{R}}_{i-1}, \underline{q}_i)$ and $\underline{r}_i = \underline{r}_i(\underline{r}_{i-1}, \underline{\mathrm{R}}_{i-1}, \underline{\mathrm{R}}_i, \underline{q}_i)$, where $\underline{q}_i$ is the subset of $\underline{q}$ corresponding to the parent-child relative transformation, $\underline{r}$ is the CM position vector and $\underline{\mathrm{R}}$ the body rotation matrix. This scheme can be exploited to find the Jacobian matrices of the transformation between absolute and generalized coordinates by automatic differentiation.

A first approach to assess the system stability is to apply classical stability analysis. In this case, each pair $(\dot{s}_Z, s_Z)$ yields a new system, which is linearized around a (hyperbolic) fixed point $\underline{q}_d^*$ found by solving $\underline{f}(\underline{0}, \underline{q}_d^*; s_Z, \dot{s}_Z) = \underline{0}$, yielding $\left[\begin{array}{cc} \dot{\underline{u}}^T & \dot{\underline{q}}^T \end{array}\right]^T = \underline{\mathrm{A}} \left[\begin{array}{cc} \underline{u}^T & \underline{q}^T \end{array}\right]^T$. The same automatic differentiation scheme can be employed to construct an exact linearization. An eigenvalue analysis of $\underline{\mathrm{A}}$ is subsequently carried out to assess the system stability, and the influence of several factors and scenarios can hereto be studied. Nonetheless, it must be noted that this approach only makes sense in singular cases, for example when the train travels along a straight line (Figure 1) or a banked curve on the horizontal plane at a constant speed. In any other scenarios, the equilibrium point looses its significance, and instead a power spectral density analysis is carried out.

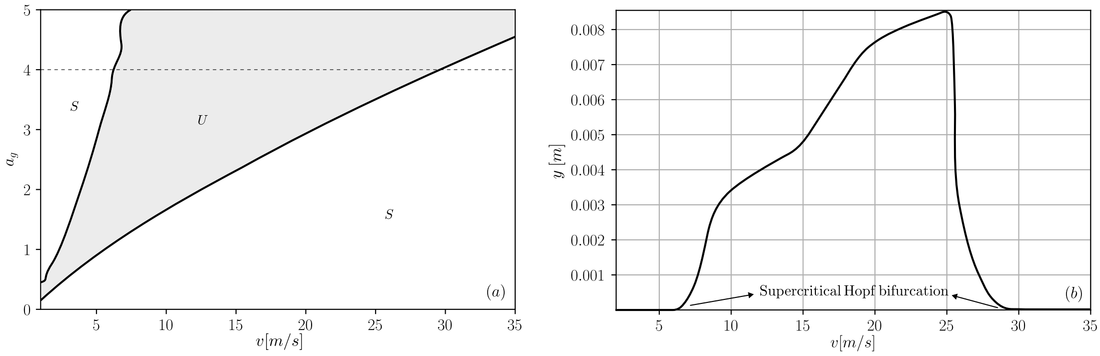

Figure 1: (a) Stability chart: (vertical g-load vs. speed) | (b) Bifurcation diagram: lateral motion vs. speed.

3 Results

Even without the presence of external excitation, various tested scenarios show different vibration levels (Figure 2), indicating that there might also be a source of self-excitation in real systems. In this case, the simulations were run without any kind of perturbation, and the ideal track trajectory itself can be thought of a soft excitation.

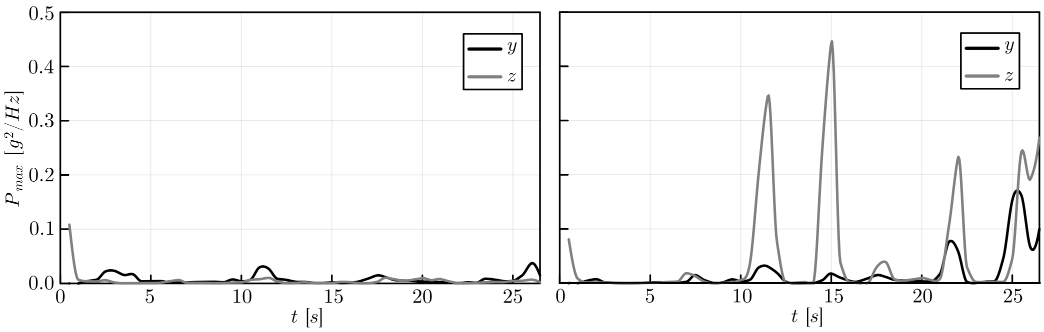

Figure 2: Maximum PSD over time (1 s window with 0.5 s overlap) of third chassis CM filtered g-loads in lateral (y) and vertical (z) directions. Case without (left) and with (right) contact friction.

4 Conclusion

The presented methodology allows for a fast analysis of the vibrational response of a roller coaster train along spatial trajectories under different conditions. For some parameter combinations, regions of increased oscillations are detected. The presented results correspond to a soft excitation, that is, due to the changing ideal trajectory itself. The next step is to study the behavior of the same system under hard excitation, namely, with locally prescribed perturbations. Finally, combining the proposed model with external excitations (trajectory deviations and local defects) is a subject of future research.

References

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