

# Further insight on the reference conditions in the Floating Frame of Reference formulation

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## EXTENDED ABSTRACT

### 1 Introduction

The reference conditions (RCs) eliminate the redundancy of the rigid motions and allow the decoupling between the gross motion of the floating frame and the elastic deformations evaluated with respect to it. The RCs have been a source of debate over the years due to their arbitrariness. Indeed, it cannot be argued that one set of RCs is better than another for every application. Arbitrariness in their choice does not translate into invariance of the results. The RCs can be interpreted as isostatic or hyperstatic constraints of a structure [1]. By binding a structure with respect to its floating frame, the rigid motions of the elastic field, linked to the description of a shape matrix, are eliminated. The RCs decouple the gross motion of the floating frame, described through translation and rotation parameters, from the elastic deformations evaluated with respect to the floating frame itself. This subdivision is only one of the infinite possibilities to decompose a flexible body's motion. For example, choosing a floating frame rigidly connected to the point of the structure, the displacement field becomes the sum of the rigid motion of the floating frame and the deformations with respect to it. Although, in this particular case, the floating frame is no longer floating; in general, the floating frame generates a motion that does not coincide with the rigid motion of the body. For this reason, one prefers to speak of gross motion rather than the rigid-body motion of the floating frame. Formally, the combination of the gross motion with the elastic deformation field should reconstruct the motion field of a flexible body regardless of the choice of the set of reference conditions. However, it has been repeatedly highlighted that choosing different conditions generates different results. Sometimes these results are very similar, or at least comparable; other times, they differ substantially. We have identified two causes behind these possible discrepancies: discretization of the finite element structure; use of component modes in reduced models.

The first cause is related to the finiteness of the elastic parameters of a discrete model concerning the infinity of deformable modes of a continuous model. Indeed, it is known that a discrete description of the elastic field leads to the stiffening of the structure. While a continuum body is free to deform following infinite shapes, any discretization forces the body to adapt to a limited number of shapes equal to the number of dof chosen in the discretization. These discrete forms are free and generally include the rigid motions of the body as well [2]. Introducing a set of RCs constrains the structure by depriving it of some dofs by modifying and reducing the shapes the body can follow to deform. Combining these elastic shapes with gross motion must allow the body to adapt to the constraints imposed by the joints that connect it to a multibody system. At this point, the flexible body may be able to satisfy the constraints imposed by the joints but produce results far from reality. The body has, in fact, adapted to the constraints by exploiting a limited set of elastic shapes, and this has compromised the results. On the other hand, the body may fail to meet constraints within a given tolerance; in this case, the kinematics constraints are violated, and the simulation is unsuccessful. This first cause is, unfortunately, inherent in any discrete numerical modeling and cannot be eliminated but is only limited through a suitable choice of reference conditions and finite elements.

The second cause follows the first because it further reduces the forms of deformation available to the body. If the number of dofs used in the discretization is too high, the resulting model will be impractical to manage in reasonable calculation times. To reduce complexity, one resorts to the use of reduced models that select only a small number of characteristic elastic shapes, called component modes. Usually, these shapes do not concern the single dof but a subset or the entire body. The reduced model obtained is certainly more computationally manageable but, at the same time, less adaptable to the constraints imposed by the joints. To alleviate these limitations, we suggest: 1) fitting the RCs to multibody joints by choosing a subset of constraints than those imposed by the joints; 2) avoiding the use of RCs that lead to hyperstatic structures; 3) trying different finite elements; 4) employ different sets of RCs to discover those that provide similar results.

### 2 Numerical results

In three papers [3, 4, 5], Shabana and his co-authors have investigated the connection between RCs and multibody joints. Considering a beam model divided into twelve identical planar beam elements connected to the frame by two pin joints at the nodes 1 and 9, Shabana demonstrated that the mean-axis RCs provide wrong results when a vertical force is applied on the beam. Considering the same pinned-pinned beam as in [4], we verified that the simply-supported RCs at nodes 1 and 9 always provide the correct results. The cantilever and simply-supported reference conditions at nodes 4 and 13 provide reliable results for small displacements, not those generated by the vertical force of 300 (N) as in [4]. While different RCs provide comparable results for small displacements, the main-axis RCs over-constrain the system. Referring to Fig. 1, the same issue raised by the pinned-pinned beam remains for a fixed-pinned beam and disappears when a roller replaces the pin joint. Observing the four cases of

Fig. 1, we conclude that the discrepancy in the results provided by the mean-axis RCs comes from the axial over-constraint generated in the pinned-pinned and fixed-pinned cases. If one of the two axial constraints is removed, all results for different reference conditions converge toward a similar solution.

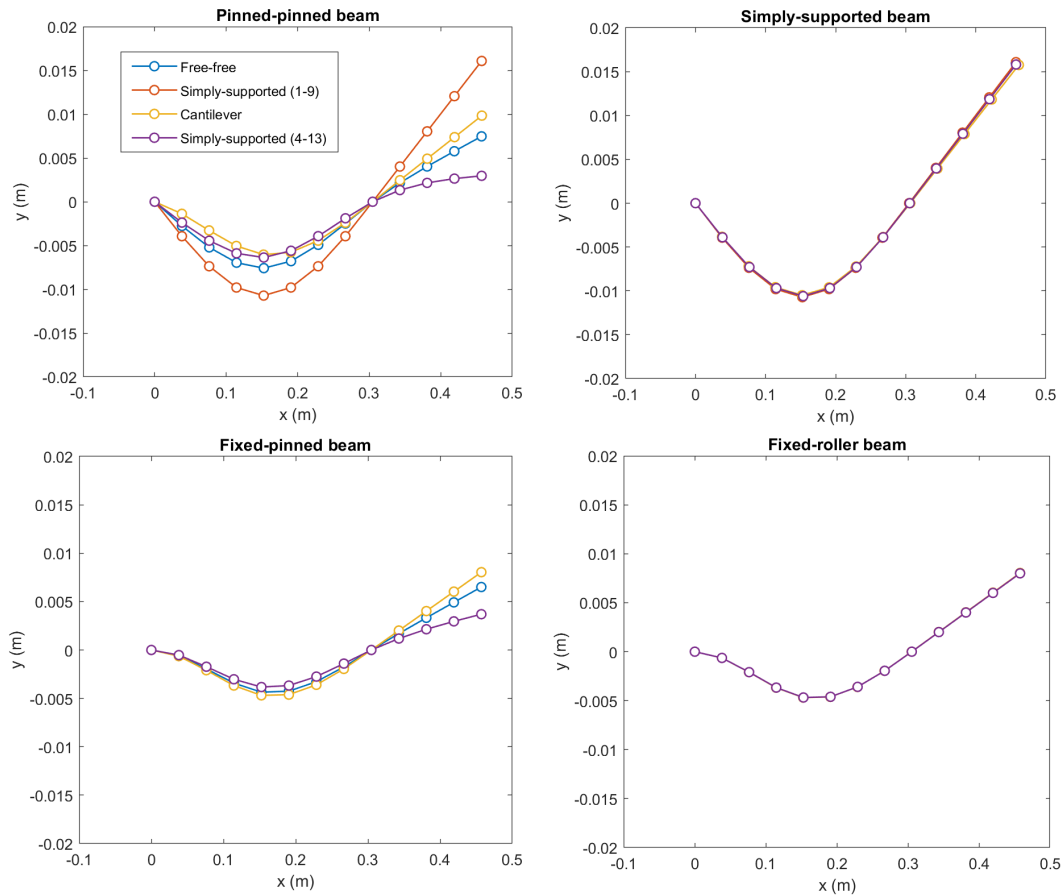


Figure 1: Static deflection of a beam with four different multibody joints and four reference conditions.

### 3 Conclusions

This work offers food for thought on a still open problem, such as the choice of the reference conditions in the Floating Frame of Reference formulation. Despite its importance, this problem still needs to be explored further by the scientific community and among multibody software developers. The correct reference conditions and their connection to multibody joints have been investigated here. We have highlighted how only the reference conditions are not suitable for every problem but must be adapted according to the constraints generated by the multibody system's joints.

### References

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