Dynamic stability of a tractor-trailer system. ZMP analysis

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ABSTRACT

Heavy vehicles have a high center of gravity. They consequently have dynamic stability problems when cornering at high speeds and when changing lanes abruptly. The most serious and most frequent accidents involve more particularly heavy articulated vehicles in traffic on freeways. The increase in the demand for safety, in particular for means of transport, but especially the enormous cost generated by this type of accident, leads to the desire to better understand the dynamic performance of these vehicles, the phenomena that could reduce their reliability, and the solutions that could improve their dynamic stability in their design. It is not sufficient to limit ourselves to solutions for the vehicle's driving rules. It is difficult for the driver to assess the stability of his vehicle, since his perception is that of the tractor's behavior, whereas it is generally the towed trailer that overturns first. This is why investigations are focused on active control systems that can act on the tipping angle, lateral load transfers, or lateral accelerations. The objective of our work is to define an active safety system to reduce the risks associated with the use of a tractor-trailer system, which focuses on the risks of lateral rollover. The equilibrium state of the system can be observed by using a stability indicator based on the position of the ZMP with respect to a polygon of sustentation that changes from one configuration to another. The proposed approach enables to verify the system along a trajectory and predict the tipping side. The rollover risk indicator has been tested and validated by simulation.

Keywords: Tractor-trailer, Dynamic stability, Lateral Rollover, Zero Moment Point.

1 INTRODUCTION

Freight transport is a vital part of the world economy and people's life. The share of road freight transport is widely significant in the world. After improving the performance and reliability of vehicles and making them accessible to a leading part of the population, more serious attention has been paid to road safety for more than half a century, given of the large number of accidents caused by this increase in the flow of people and goods transported and the terrible consequences they entail. Heavy duty vehicles occupy an increasingly significant place in our society, and pose more and more serious problems in terms of road safety. The latter relies on a fragile balance between vehicles and drivers' behavior, the road, and its environment. This issue is currently being examined and involves many scientific fields. Granting to a report published in 2018 by the World Health Organization, there are 1.35 million deaths per year. The report demands much more action by governments and partners to enact life-saving mechanisms around the world. Among accidents involving the use of a vehicle, lateral rollover is one of the most frequent and most dangerous for vehicles with an exceptionally high center of gravity. This is typically the case for tractors with trailers. Statistical studies conducted by researchers worldwide have highlighted this problem and the entire profession is currently proactively seeking solutions to reduce the consequences of this type of accident.

The study consists of predicting the rollover risk of a tractor with a trailer. The goal is to determine the most effective strategy to detect the warning signs of this type of accident using the Zero Moment Point (ZMP) approach.

2 DYNAMIC STABILITY OF A TRACTOR WITH A TRAILER

The static and dynamic stability of a tractor with a trailer is of major importance for the safety of transporting goods and people. The simulation of a tractor-trailer system and the analysis of the static and dynamic behavior is an efficient approach to enhance the safety of both operators and people around the job area. It is proposed to start with an overview of the dangerous criteria applied to road vehicles for which literature is important. Different criteria for predicting rollover are listed, delimiting their scope of application and comparing them. In all the criteria, the following factors are involved to a greater or lesser degrees: the characteristics of the road (slope, surface, inclination), the characteristics of the tractor and the trailer (kinematics, dimensions, masses, available measurements). The research conducted on the determination of analytical criteria for predicting lateral rollover can be divided into two categories. The first will use a model of the tractor-trailer system and its quasi-static behavior, the second one will involve a dynamic model of the system. In both cases, the problem related to the measurable or observable data will remain the same and will give rise to model adaptations. The criteria are developed in order to have a picture of the vehicle's dynamic stability which can be defined as the fact that the vehicle returns to its equilibrium state when the disturbances cease.

Static stability criteria are based on static or "quasi-static" modeling, they very often depend on the geometrical parameters of the vehicle. They are relevant in many practical cases and are easy to implement (Static Stability Factor [3], Side-Pull Ratio [4], Critical Sliding Velocity [5], etc.).

In dynamic approaches, the disbonding of the two wheels located on the same side of a vehicle remains an interesting starting point for the calculation of lateral stability criteria. The observation of the vertical forces exerted on the tires of the same axle becomes essential (Lateral Load Transfert [6], Dynamic Stability Index [7], Rollover Stability Advisor [8], Zero Moment Point [9, 10, 11, 12], Stability Moment [1, 13], etc.).

We have presented a list of stability criteria for rollover and their corresponding target values. The scope of this literature review is mainly restricted to road vehicles and we will study its transposition to articulated vehicles.

3 DYNAMIC MODELING

The studied vehicle is a unicycle mobile robot with a trailer (Figure 1). The unicycle mobile robot is linked to the ground via two driving wheels (equipped with actuators) and two free-caster wheels (not shown in Figure 1) to assure the static stability of the platform. The trailer is linked to the ground via two passive wheels.

In the case that the tractor is a unicycle mobile robot, the rotation movement of the system is made by the difference in the rotational speeds of the driving wheels. Moreover, the wheels are generally considered rigid, which gives the possibility to calculate the wheel/ground interaction forces by including the constraints of rolling and pivoting without slipping the wheels on the ground, in the motion equations of the system. The modeling steps of the dynamic behavior of a unicycle mobile robot with a trailer are the following: i) geometrical description of the system; ii) generation of the constraints equations, and iii) generation of the motion equations.

In order to establish the different models of this system, we note $\Re_0\,(O_0,\, \vec{x}_0,\, \vec{y}_0,\, \vec{z}_0)$ the fixed reference frame attached to the ground, $\Re_r\,(O_r,\, \vec{x}_r,\, \vec{y}_r,\, \vec{z}_r)$ the mobile reference frame attached to the tractor chassis, and $\Re_t\,(O_t,\, \vec{x}_t,\, \vec{y}_t,\, \vec{z}_t)$ the mobile reference frame attached to the trailer chassis. The generalized coordinates vector of the system is defined by :

$$\vec{q} = \begin{bmatrix} x_r & y_r & \theta_r & x_t & y_t & \theta_t & \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{bmatrix}^t \tag{1}$$

With $(x_r,\, y_r,\, \theta_r)$ and $(x_t,\, y_t,\, \theta_t)$ are the position and orientation coordinates, respectively, of the references frames $\Re_r$ and $\Re_t$ with respect to the reference frame $\Re_0$, while the φ_i are the wheel

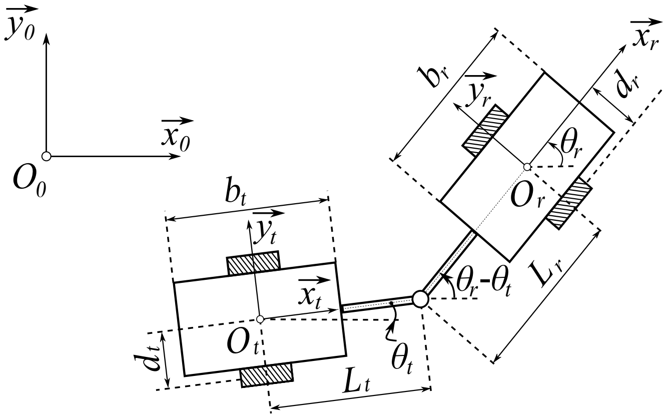


Figure 1: Unicycle mobile robot with a trailer [1].

rotation angles of the tractor and the trailer.

A tractor with a trailer is subject to holonomic and non-holonomic constraints. The holonomic constraints are defined by the conditions of the pivot articulation between the tractor and the trailer:

$$\begin{cases} x_r - L_r \cdot \cos(\theta_r) - x_t - L_t \cdot \cos(\theta_t) \\ y_r - L_r \cdot \sin(\theta_r) - y_t - L_t \cdot \sin(\theta_t) \end{cases} \tag{2}$$

Non-holonomic constraints are defined by the conditions of rolling and pivoting without slipping the wheels on the ground:

$$\begin{cases} \dot{x}_r \cdot \cos(\theta_r) + \dot{y}_r \cdot \sin(\theta_r) + d_r \cdot \dot{\theta}_r - r_w \cdot \dot{\varphi}_1 = 0 \\ \dot{x}_r \cdot \cos(\theta_r) + \dot{y}_r \cdot \sin(\theta_r) - d_r \cdot \dot{\theta}_r - r_w \cdot \dot{\varphi}_2 = 0 \\ -\dot{x}_r \cdot \sin(\theta_r) + \dot{y}_r \cdot \cos(\theta_r) = 0 \\ \dot{x}_t \cdot \cos(\theta_t) + \dot{y}_t \cdot \sin(\theta_t) + d_t \cdot \dot{\theta}_t - r_w \cdot \dot{\varphi}_3 = 0 \\ \dot{x}_t \cdot \cos(\theta_t) + \dot{y}_t \cdot \sin(\theta_t) - d_t \cdot \dot{\theta}_t - r_w \cdot \dot{\varphi}_4 = 0 \\ -\dot{x}_t \cdot \sin(\theta_t) + \dot{y}_t \cdot \cos(\theta_t) = 0 \end{cases} \tag{3}$$

To establish the motion equations, we use the Lagrangian formalism which gives the differential equations shown in Bouzar Essaidi et al [1].

$$\begin{cases} m_r \cdot \ddot{x}_r - m_r \cdot \ddot{\theta}_r \cdot x_{G_r} \cdot s_r - m_r \cdot \dot{\theta}_r^2 \cdot x_{G_r} \cdot c_r = (\lambda_1 + \lambda_2) \cdot c_r - \lambda_3 \cdot s_r + \lambda_7 \\ m_r \cdot \ddot{y}_r + m_r \cdot \ddot{\theta}_r \cdot x_{G_r} \cdot c_r - m_r \cdot \dot{\theta}_r^2 \cdot x_{G_r} \cdot s_r = (\lambda_1 + \lambda_2) \cdot s_r + \lambda_3 \cdot c_r + \lambda_8 \\ \left(I_{z_r} + m_r \cdot {x_{G_r}}^2\right) \cdot \ddot{\theta}_r + m_r \cdot x_{G_r} (-\ddot{x}_r \cdot s_r + \ddot{y}_r \cdot c_r) = d_r \cdot (\lambda_1 - \lambda_2) - L_r \cdot \lambda_8 \cdot c_r + L_r \cdot \lambda_7 \cdot s_r \\ I_{y_w} \cdot \ddot{\varphi}_1 = \tau_1 - r_w \cdot \lambda_1 \\ I_{y_w} \cdot \ddot{\varphi}_2 = \tau_2 - r_w \cdot \lambda_2 \\ m_t \cdot \ddot{x}_t - m_t \cdot \ddot{\theta}_t \cdot x_{G_t} \cdot s_t - m_t \cdot \dot{\theta}_t^2 \cdot x_{G_t} \cdot c_t = (\lambda_4 + \lambda_5) \cdot c_t - \lambda_6 \cdot s_t - \lambda_7 \\ m_t \cdot \ddot{y}_t + m_t \cdot \ddot{\theta}_t \cdot x_{G_t} \cdot c_t - m_t \cdot \dot{\theta}_t^2 \cdot x_{G_t} \cdot s_t = (\lambda_4 + \lambda_5) \cdot s_t + \lambda_6 \cdot c_t - \lambda_8 \\ \left(I_{z_t} + m_t \cdot {x_{G_t}}^2\right) \cdot \ddot{\theta}_t + m_t \cdot x_{G_t} \cdot (-\ddot{x}_t \cdot s_t + \ddot{y}_t \cdot c_t) = d_t \cdot (\lambda_4 - \lambda_5) + L_t \cdot (\lambda_7 \cdot s_t - \lambda_8 \cdot c_t) \\ I_{y_w} \cdot \ddot{\varphi}_3 = -r_w \cdot \lambda_4 \\ I_{y_w} \cdot \ddot{\varphi}_4 = -r_w \cdot \lambda_5 \end{cases} \tag{4}$$

$c_r = \cos(\theta_r)$, $s_r = \sin(\theta_r)$, $c_t = \cos(\theta_t)$, and $s_t = \sin(\theta_t)$; m_r (resp. m_t) et I_{z_r} (resp. I_{z_t}) are the total mass and the vertical moment of inertia of the tractor (resp. trailer), respectively; $(x_{G_r},\ 0,\ z_{G_r})^t$ (resp. $(x_{G_t},\ 0,\ z_{G_t})^t$) are the coordinates of the center of gravity of the tractor (resp. trailer); I_{y_w} is the moment of inertia of the wheel; r_w is the radius of the wheels; λ_i, for $i = 1...8$, are the Lagrange multipliers related to the holonomic and non-holonomic constraints; τ_1 and τ_2 are the torques of the actuators applied by the tractor wheels.

4 EVALUATION OF THE ZMP

This study detects the precursor signs of lateral rollover for the tractor-trailer system. We propose to use the positions of the wheel/ground contact points to determine the possible polygon(s) of sustentation and then to locate the ZMP concerning this set of polygons of sustentation. This analysis allows us to define a dynamic stability criterion. This criterion is very often applied in humanoid robotics [2] to diagnose the state of equilibrium of a multi-body system with a modifiable configuration. We propose to adapt its use to the case of a tractor with a trailer. For this purpose, we use the criterion related to the calculation of the ZMP to establish an indicator of the risk of rollover.

To evaluate the position of the ZMP, it is necessary, first of all, to apply the fundamental principle of dynamics to the tractor-trailer system. This principle applied to the system (Σ), expressed in the reference frame $\Re_0$, at point G is given by :

$$\left\{ \begin{array}{c} m \cdot \vec{\Gamma}_{G\in\Sigma/R_0} \\ \vec{\delta}_{G,\Sigma/R_0} \end{array} \right\}_{R_0} = \left\{ \begin{array}{c} \overrightarrow{\Re}_{\text{out}} \\ \vec{M}_{G,\Re_{out}\rightarrow\Sigma/R_0} \end{array} \right\}_{R_0} \tag{5}$$

With :

- $\vec{\Gamma}_{G\in\Sigma/R_0}$ is the linear acceleration of the point (G), with respect to the reference frame $\Re_0$.
- $\vec{\delta}_{G,\Sigma/R_0}$ is the dynamic moment, calculated at point (G), with respect to the reference frame $\Re_0$.
- $\overrightarrow{\Re}_{\text{out}}$ is the resultant of the external forces applied to the system.
- $\vec{M}_{G,\Re_{out}\rightarrow\Sigma/R_0}$ is the moment of the external forces applied and calculated at point G.

For the evaluation of the position of the ZMP, we will be interested only in the equation of the moments of the fundamental principle of dynamics which must be calculated at O_{ZMP}. By definition, the ZMP belongs to the ground. The determination of the ZMP position will be carried out by allowing only the yaw rotation around the normal to the ground.

The dynamic moment of the tractor is calculated at point G_r as follows :

$$^{R_r}\vec{\delta}_{G_r,S_r/R_0} = [I_{G_r,S_r}] \left[\frac{d}{dt} \vec{\Omega}_{R_r/R_0} \right] \tag{6}$$

The dynamic moment of the trailer is calculated at point G_t as follows :

$$^{R_t}\vec{\delta}_{G_t,S_t/R_0} = [I_{G_t,S_t}] \left[\frac{d}{dt} \vec{\Omega}_{R_t/R_0} \right] \tag{7}$$

Using the fundamental principle of dynamics on the tractor-trailer system, we find the moment at point O_{ZMP} and expressed in $\Re_r$:

$$[^rP_t]^{R_t}\vec{\delta}_{G_t,S_t/R_0} + [^rP_r]^{R_r}\vec{\delta}_{G_r,S_r/R_0} + {}^r\overrightarrow{O_{ZMP}G_t} \times {}^r\overrightarrow{F}_{G_t\in S_t/R_0} + {}^r\overrightarrow{O_{ZMP}G_r} \times {}^r\overrightarrow{F}_{G_r\in S_r/R_0} = {}^r\vec{M}_{ZMP} \tag{8}$$

The forces acting on each platform were calculated and expressed in the reference frame $\Re_r$ linked to the tractor :

$$
{}^{r}\overrightarrow{F}_{G_r \in S_r/R_0} = m_r \cdot {}^{r}R_0 \cdot \left(\overrightarrow{g} + \overrightarrow{\Gamma_r}\right) = \begin{pmatrix} m_r \cdot (c_r \cdot \ddot{x}_r + s_r \cdot \ddot{y}_r) \\ m_r \cdot (-s_r \cdot \ddot{x}_r + c_r \cdot \ddot{y}_r) \\ m_r \cdot g \end{pmatrix} \tag{9}
$$

$$
{}^{r}\overrightarrow{F}_{G_t \in S_t/R_0} = m_t \cdot {}^{r}R_0 \cdot \left(\overrightarrow{g} + \overrightarrow{\Gamma_t}\right) = \begin{pmatrix} m_t \cdot (c_r \cdot \ddot{x}_t + s_r \cdot \ddot{y}_t) \\ m_t \cdot (-s_r \cdot \ddot{x}_t + c_r \cdot \ddot{y}_t) \\ m_t \cdot g \end{pmatrix} \tag{10}
$$

The coordinates of the ZMP relative to the centers of gravity of the tractor and trailer are :

$$
{}^{r}\overrightarrow{O_{ZMP}G_r} = {}^{r}R_0 \cdot \begin{pmatrix} x_r - x_{ZMP} \\ y_r - y_{ZMP} \\ -z_r \end{pmatrix} = \begin{pmatrix} c_r \cdot (x_{ZMP} - x_r) + s_r \cdot (y_{ZMP} - y_r) \\ -s_r \cdot (x_{ZMP} - x_r) + c_r \cdot (y_{ZMP} - y_r) \\ -z_r \end{pmatrix} \tag{11}
$$

$$
{}^{r}\overrightarrow{O_{ZMP}G_t} = {}^{r}R_0 \cdot \begin{pmatrix} x_t - x_{ZMP} \\ y_t - y_{ZMP} \\ -z_t \end{pmatrix} = \begin{pmatrix} c_r \cdot (x_{ZMP} - x_t) + s_r \cdot (y_{ZMP} - y_t) \\ -s_r \cdot (x_{ZMP} - x_t) + c_r \cdot (y_{ZMP} - y_t) \\ -z_t \end{pmatrix} \tag{12}
$$

The dynamic moments of each mobile platform are expressed, in the reference frame $\Re_r$, as follows :

$$
{}^{R_r}\vec{\delta}_{G_r,S_r/R_0} = \begin{pmatrix} 0 \\ 0 \\ I_{z_r} \cdot \ddot{\theta}_r \end{pmatrix} ; {}^{R_r}\vec{\delta}_{G_t,S_t/R_0} = {}^{r}R_0 \cdot {}^{0}R_t \cdot {}^{R_t}\vec{\delta}_{G_t,S_t/R_0} = \begin{pmatrix} 0 \\ 0 \\ I_{z_t} \cdot \ddot{\theta}_t \end{pmatrix} \tag{13}
$$

By annulling the components of the dynamic moment of the tractor-trailer system calculated at point O_{ZMP} :

$$
\left\{ \begin{array}{l} {}^{r}\vec{M}_{ZMP} \cdot \vec{x}_r = 0 \\ {}^{r}\vec{M}_{ZMP} \cdot \vec{y}_r = 0 \end{array} \right\} \Rightarrow [\mathrm{A}] \cdot \overrightarrow{X} = \overrightarrow{B} \tag{14}
$$

Here, $[\mathrm{A}] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$; $\overrightarrow{X} = \begin{pmatrix} x_{ZMP} \\ y_{ZMP} \end{pmatrix}$; $\overrightarrow{B} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$

With, $A_{11} = -(m_r + m_t) \cdot g \cdot s_r$, $A_{12} = (m_r + m_t) \cdot g \cdot c_r$, $A_{21} = -(m_r + m_t) \cdot g \cdot c_r$,
and $A_{22} = -(m_r + m_t) \cdot g \cdot s_r$

$$
B_1 = -g \cdot s_r \cdot (m_r x_r + m_t x_t) + g \cdot c_r \cdot (m_r y_r + m_t y_t) - h_r \cdot m_r \cdot (-s_r \ddot{x}_r + c_r \ddot{y}_r) - h_t \cdot m_t \cdot (-s_r \ddot{x}_t + c_r \ddot{y}_t)
$$

$$
B_2 = -g \cdot c_r \cdot (m_r x_r + m_t x_t) - g \cdot s_r \cdot (m_r y_r + m_t y_t) + h_r \cdot m_r \cdot (c_r \ddot{x}_r + s_r \ddot{y}_r) + h_t \cdot m_t \cdot (c_r \ddot{x}_t + s_r \ddot{y}_t)
$$

h_r (resp. h_t) is the height of the center of gravity of the tractor (resp. trailer).

5 PREDICTION OF THE SITUATION

A criterion of dangerousness is interesting to prevent an operator that a situation is dangerous whereas this one, occupied with another task, could not be aware of it. The pertinence of a danger criterion is all the more important as this information allows us to avoid dangerous situations. From the evaluation of the ZMP position, it can be seen that the tractor-trailer system tends to "take off" from the ground or to get closer to it. To visualize the equilibrium of a tractor with a trailer, it is possible to construct a dynamic stability indicator of the system using the ZMP. The concept consists in evaluating the stability of the system by using the position of the ZMP relative to the polygon of sustentation. We say that the system is stable when the position of the ZMP remains inside the polygon of sustentation, that it is at the limit of stability if the ZMP is at the border of the polygon, and that the tractor-trailer system is at the limit of overturning laterally on the left or on the right side, and that if not, it is not stable because it risks tipping.

In the case of a tractor with a trailer, the geometric shape of the polygon of sustentation (Figure 2.a) differs from one configuration to another. This makes it difficult for some methods to analyze dynamic stability. For this reason, the criterion related to the calculation of the ZMP is used to establish an indicator of rollover risk. The evaluation of the dynamic stability is performed by comparing the following two surfaces:

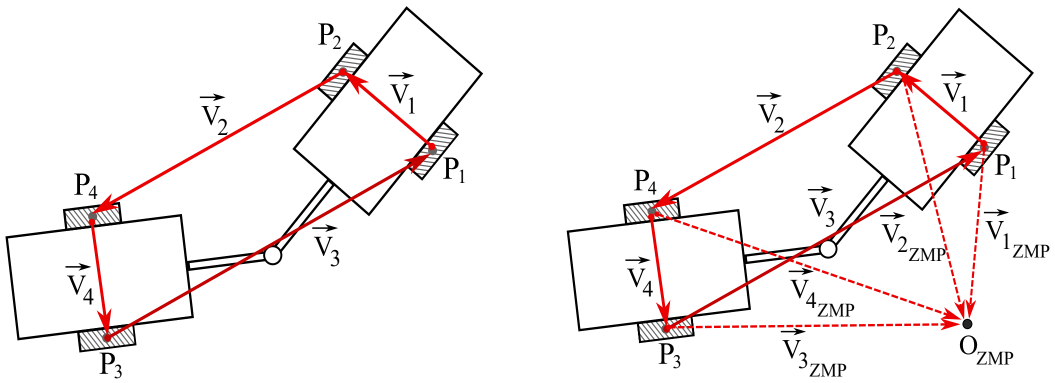
$S_{Poly} = S_1 + S_2$ and $S_{ZMP} = \sum_{i=1}^{4} A_i$.

With, $S_1 = \frac{\left|\vec{V_1}\times\vec{V_2}\right|}{2}$, $S_2 = \frac{\left|\vec{V_3}\times\vec{V_4}\right|}{2}$, and $A_i = \frac{\left|\vec{V_i}\times\overrightarrow{V_{i_{ZMP}}}\right|}{2}$, for $i = 1 \dots 4$.

The dynamic stability index of a tractor with a trailer is given by :

$$R_{ZMP} = S_{ZMP} - S_{Poly} \tag{15}$$

Note that R_{ZMP} is a dimensionless function that takes two positive values if the ZMP is outside the polygon of sustentation (Figure 2.b). In particular, if $R_{ZMP} = 2 \cdot A_3$, the tractor-trailer system is reversed to the right side and if $R_{ZMP} = 2 \cdot A_2$, the system is reversed to the left side. Whereas, the tractor-trailer system is stable if $R_{ZMP} = 0$.



(a) ZMP inside the polygon.

(b) ZMP outside the polygon.

Figure 2: Evaluation of the dynamic stability index.

6 RESULTS

This section proposes a comparison of the results given by the ZMP-based stability criterion with those given by the Stability Moment Method (SM) based criterion [1], in the case of a tractor with a trailer. The geometric and inertial parameters of the tractor and trailer are shown in Table 1.

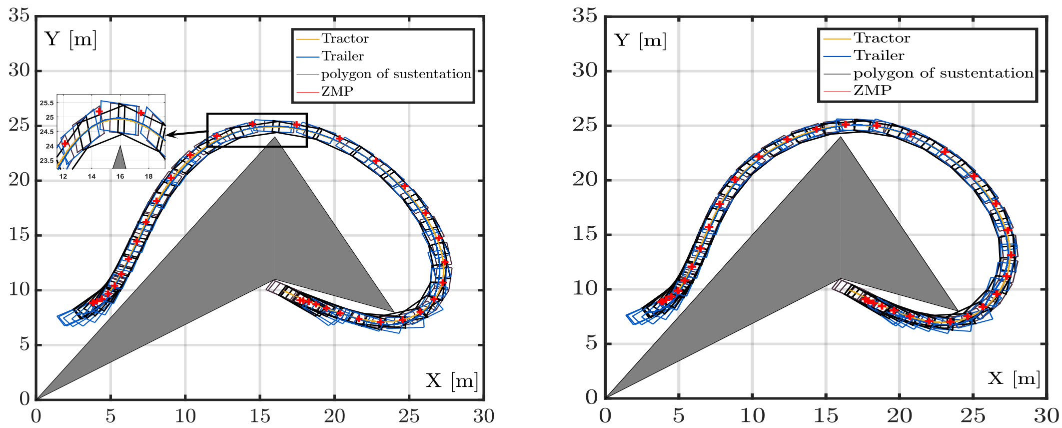
The problem addressed in this reference [1], is to test the performance of the SM method when dynamic stability is accounted for along a trajectory planned in minimum time.

The first solution shown in Figure 3.a was computed without taking into account the dynamic stability. The stability index R_{SM} is not kept within the authorized limits ($-1 < R_{SM}(t) < 1$), also the stability index R_{ZMP} is outside the polygon of sustentation ($R_{ZMP} > 0$). By evidence, if the tractor-trailer system tracks this trajectory, it will be overturned to the left side which coincides with a right turn ($R_{ZMP} = 2 \cdot A_2$).

b_r $[m]$	b_t $[m]$	L_r $[m]$	L_t $[m]$	d_r $[m]$	d_t $[m]$	r_w $[m]$
2.0	2.5	1.5	1.75	0.5	0.5	0.3
x_{G_r} $[m]$	x_{G_t} $[m]$	m_r $[kg]$	m_t $[kg]$	I_{z_r} $[kg.m^2]$	I_{z_t} $[kg.m^2]$	I_{y_w} $[kg.m^2]$
0.25	0.25	54.0	74.0	26.4	32.4	0.006

Table 1: Tractor and trailer parameters [1].

The second solution shown in Figure 3.b was computed for the same problem but taking into account the dynamic stability. The stability indices R_{SM} and R_{ZMP} remain within the admissible limits ($|R_{SM}| < 1$ and $R_{ZMP} = 0$) and the ZMP is located inside the polygon of sustentation in each configuration.



(a) Without dynamic stability. (b) With dynamic stability.

Figure 3: Evaluation of the dynamic stability index.

The example discussed in this section shows that the proposed approach achieves competitive results.

7 CONCLUSION

Lateral rollover remains one of the most frequent and dangerous accidents related to the use of tractor-trailer system. The developed work is a contribution to the stability diagnosis, in the sense of non-rollover, and the balance of a tractor-trailer system. We have provided solutions to this problem by using the criterion associated with the "Zero Moment Point (ZMP)". The study focuses on dynamic modeling of a tractor, of unicycle-type, with a trailer as well as the estimation of the ZMP position, as a dynamic stability criterion, which is used to build an indicator of rollover risk (instability).

The technique applied to evaluate and quantify the dynamic stability of the tractor-trailer system gave us the opportunity to obtain information states, check the system along a trajectory, and predict the tipping side, while avoiding the risks of overturning and loss of equilibrium. We envisage, as future work, to realize experimental tests on a tractor with a trailer.

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