Physical Validation of Simulation Tools for Slender Elastic Structures

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EXTENDED ABSTRACT

1 Introduction

This contribution considers physical validation for both industrial and research simulation tools. Romero et. al [1] recently proposed a framework consisting of four benchmark tests for physical validation of simulation tools for slender elastic structures in computer graphics. We apply the rod and shell models implemented in the simulation tool IPS (Industrial Paths Solutions) [2, 3], the CRod model [4], and ODIN [5] to three of the benchmark tests illustrated by exemplary equilibrium states in Fig. 1. These models are particularly designed for efficient (in the case of IPS real-time) simulations and thus share the motivational aspects prevailing in computer graphics.

We show that for sufficient precision in meshes and load steps all simulations meet the expected results based on the master curves from [1] – in particular for the parameter ranges which are relevant to the application field. In this course, we study the numerical behavior of the models behind the software employed and the benchmarks.

2 Cantilever Bending

The first benchmark is valid for testing both shell and rod models, meaning structures that are thin in one or two directions, respectively. The object of interest is clamped at one end (for the shell such that the direction of gravity points in the thickness dimension) and free on the other end. When then exposed to gravity, the object bends downwards as displayed for the rod in Fig. 1a. In this way, the total coordinate ratio $\frac{\Delta y}{\Delta x}$ measured between the end points may be plotted against the dimensionless gravito-bending parameter

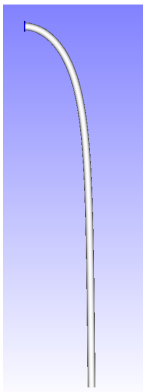
$$\Gamma_{rod} = \frac{\rho A_{\circ} g L^3}{EI} \qquad \Gamma_{shell} = \frac{\rho A_{\square} g L^3}{Dw}$$

for geometrical parameters length L, shell width w, cross-section areas A, and second moment of area I, and mechanical parameters density ρ, Young's modulus E, flexural rigidity D, and gravity g.

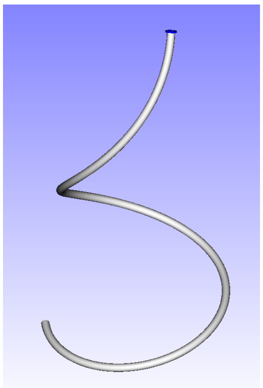
Figure 2 displays the semi-analytic master curve and the simulation results for the IPS beam and shell on a logarithmic scale. For rather coarse discretizations (e.g. 15 elements in the length dimension for the shell) and high values $\Gamma > 5e3$ the simulations deviate from the master curve, but compute the physically correct solutions for lower values or finer discretizations. Notably, the range of meaningful magnitudes of Γ for simulation of circular and flat cables is $[10^0, 10^2]$ and $[10^0, 10^3]$, respectively.

3 Bend-Twist Bifurcation

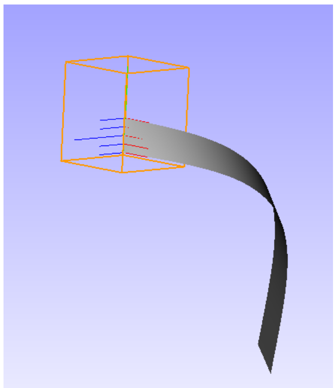
For rods the coupling of bending and twist gives rise to several phenomena which are demanding to capture by simulations. We clamp a naturally circular rod vertically at one end, such that the initial tangent points in the direction of gravity. Depending on



(a) Cantilever bending (rod)



(b) Bend-twist bifurcation



(c) Lateral buckling

Figure 1: Exemplary equilibrium configurations for the three benchmark examples.

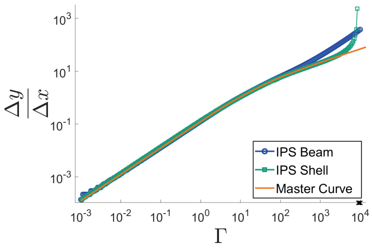


Figure 2: Overview of Cantilever benchmark computed for the IPS cable segment (rod) and the IPS flat cable (shell). The green and blue markers are aligned accurately along the orange master curve for the relevant intervals.

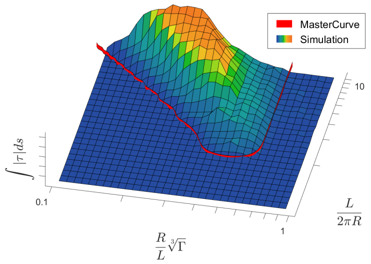


Figure 3: Integrated Frenet torsion of the equilibria in the Bend-Twist benchmark in IPS for a coarse discretisation. Two-dimensional configurations are characterized by this integral being zero (dark blue region). Ground truth determined by the master curve displayed as red wall.

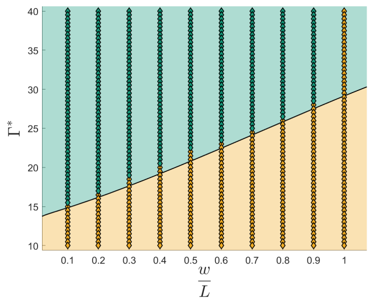


Figure 4: Overview of Lateral Buckling benchmark in IPS for a shell with 20 elements in length dimension. Ground truth determined by master curve displayed in the background, simulations shown as diamonds. Three-dimensional configurations in green and stable two-dimensional states in orange.

both the relationship between length L and bending radius R, and the gravito-bending parameter Γ either the naturally planar state is a stable equilibrium or a buckling to a three-dimensional state such as in Fig. 1b occurs.

We observe a numerically very challenging behavior of the Bend-Twist benchmark. First, the circular reference configuration requires a relatively fine discretization. Second, IPS does not feature perturbing the reference in terms of curvatures as described in [1] such that we decide for rotating the direction of gravity slightly forwards and backwards again. Third, distinction between plane and spacial case involves discrete approximation of the Frenet torsion (which is identically zero for plane curves), numerical integration and choice of a threshold. Figure 3 displays the integrated absolute torsion in a height map for different test samples varying in bending radius and gravity which quantitatively well fits together with the master curve.

4 Lateral Buckling

When clamping a ribbon such that its width dimension equals the direction of gravity, the gravito-bending parameter Γ and the aspect ration of width to length $\frac{w}{L}$ decide whether the planar state is a stable equilibrium or a lateral buckling occurs at the slightest perturbation as depicted in Fig. 1c.

Figure 4 shows that the shell model in IPS can compute the bifurcation point quantitatively well even for rather coarse discretizations (e.g. 20 elements in length dimension), as long as $w < L$. Computations show, that for fine discretizations (e.g. 45 elements in length dimension) the master curve is captured accurately. Naturally, distinct choices of geometry and material parameters lead to different conditioning in the numerics and thus to significant variations of the shell behavior.

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