Uncertainty analysis of a contact-based multibody model of meshing spur gears

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EXTENDED ABSTRACT

1 Introduction

The adoption of numerical methods to simulate complex physical phenomena has improved significantly in recent decades, thanks to the fast growth of computing hardware and algorithm developments. Numerical methods are based on a simplified physical (and consequently mathematical) model able to reproduce with a certain error the real physical phenomena. The accuracy of the model always depends on the level of approximations adopted and consequently on the uncertain input of the model. Gear dynamics is one of the most discussed topics in engineering because gears are the most common components adopted for power transmission. For this reason, many researchers are actively working to develop sophisticated models for gear dynamic simulations capable of identifying vibrations generated by teeth impacts, optimizing the design process, and improving the current generation of diagnostic techniques. It has been demonstrated [1] that the variation of mesh stiffness, depending on the number of contact teeth and the flexibility of the gear, becomes a source of vibration in dynamic conditions. A meaningful parameter to evaluate this excitation is the transmission error (*TE*).

$$TE(t) = \theta_1(t) r_{b1} + \theta_2(t) r_{b2} \tag{1}$$

The methods adopted for gear simulation can be divided into four families: experiment method (EM), finite element method (FEM), traditional analytical method (AM) [2, 3], and multibody dynamics methods (MUBO). FEs methods are accurate, but they lack generality and are very computationally demanding; AMs are hard to set because they need a complex pre-processing phase; MUBO represent a good compromise between computational time and reliability. In MUBO the gears are generally considered as rigid bodies and the contact between the involute profiles is established through a detection method. Consistent with the approach in [4], the contact force generated at the contact point is based on a penalty contact force. The contact force can be calculated using the following relationship.

$$F_n = k_{con}\delta^{m_1} + c_{con}\frac{\dot{\delta}}{\left|\dot{\delta}\right|}\left|\dot{\delta}\right|^{m_2}\delta^{m_3} \tag{2}$$

Thanks to the multibody approach, it is possible to effectively represent the evolution of contact points along the profile, the friction of the surfaces, and it is also possible to assess the dynamic of the system under several operating conditions, such as variable torque or acceleration. In [5] a family of MUBO contact-based models able to consider the compliance of teeth through pseudo-rigid approach (MUBOCO-PR) has been introduced and compared. MUBOCO-PR is essentially a pseudo-rigid multibody system in which the teeth are considered as rigid bodies connected to the main body (the gear foundation) through specific joints located in the dedendum circle and an equivalent spring (Figure 1). In [6, 7] it has been demonstrated the high-accuracy of the MUBOCO-PR in the evaluation of the transmission error both in static and dynamic conditions.

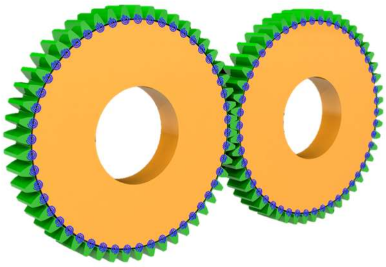

Figure 1: The multibody model with a revolute joint and a rotational spring. [8]

As in all simplified models, even the MUBOCO-PR requires an *a priori* identification of the lumped parameters. In [6] a reliable approach to identify these parameters is provided. However, a systematic approach to estimate the uncertainty of the model is not already discussed. In this study, an evaluation of the uncertainty of the MUBOCO-PR model is carried out using the fuzzy arithmetic-based transformation method [9]. The transformation method proposed by Hanss [10], describes the lack of certainty of the input parameters as fuzzy numbers and provides a simple but very powerful tool for estimating the uncertainties of the output as fuzzy-valued quantities. This method is a very practical application of fuzzy arithmetic to solve complex engineering problems. However, the disadvantage of the transformation method is that it usually requires many physical models to be evaluated. Furthermore, the number of evaluations increases exponentially with the number of uncertain input parameters. In this paper, the fuzzy-based method is applied to evaluate the effects of the epistemic uncertainty of the MUBOCO-PR model. In particular, first, the uncertainty of the stiffness of the rotational springs and the contact stiffness along the meshing line is considered. Both inputs are defined as triangular fuzzy numbers in which the modal values are analytically defined. The application of the transformation method with six cuts along the membership functions and seven sampling points each resulted in 49 quasistatic multibody simulations to evaluate the static transmission error (STE) of the gears. In Figure 2 (left) the results of the STE function of the mesh period computed using this approach are reported and compared to FEM solution (black line with cross markers).

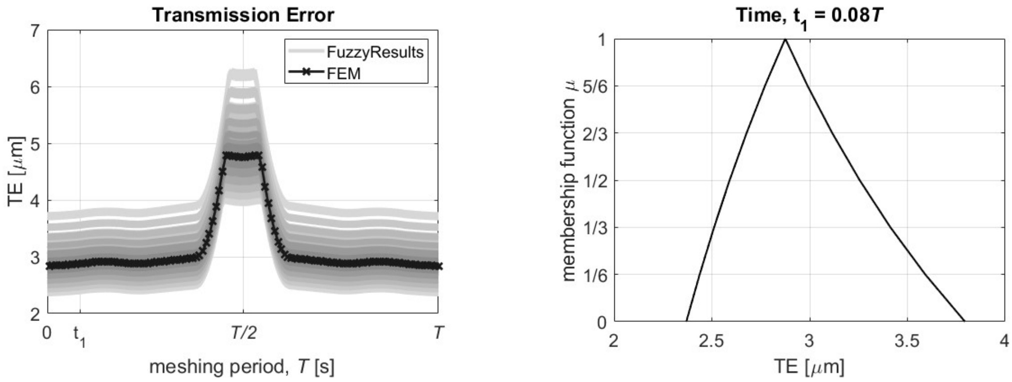

Figure 2: on the left: STE of the models simulated; on the right: Fuzzy output number at t_1=0.08T.

In Figure 2 (right) the fuzzy number of the STE is shown at a selected meshing time (t_1). Due to the nonlinearity of the system, the output fuzzy number does not have the same triangular shape as the input parameters and the modal value is not centered with regard to the support obtained. As the membership function, *i.e.* the degree of input certainty, decreases, the asymmetry increases. The use of this method makes it feasible to accurately assess the contribution of each model parameter's level of uncertainty to the overall degree of uncertainty of the model output.

Acknowledgments

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