A Quasistatic Contact Model for Impact Analysis in Flexible Multibody Systems Based on IGA

Tobias Rückwald, Alexander Held, Robert Seifried

Institute of Mechanics and Ocean Engineering
Hamburg University of Technology
Eißendorfer Straße 42, 21073 Hamburg, Germany
[tobias.rueckwald,alexander.held,robert.seifried]@tuhh.de

EXTENDED ABSTRACT

1 Introduction

The focus of this work is an efficient but detailed simulation of impacts in flexible multibody systems. Thereby the floating frame of reference formulation is used [1], which requires global shape functions of the flexible bodies. Usually, a finite element model consisting of isoparametric elements is used to approximate the global shape functions. A disadvantage of isoparametric elements is that the geometry is discretized. However, impact simulations depend on an accurate representation of the geometry in the contact area. As an alternative approach, isogeometric elements can be used where there is no error in the representation of the geometry. For this reason, the isogeometric analysis (IGA) will be employed in this work to determine the global shape functions. In order to preserve the local deformation of the contact region in the model reduction, a Craig-Bampton method is used. In a previous work [2], the IGA bodies are reduced with a Craig-Bampton method resulting in numerically stiff equations of motion despite additionally added numerical damping. The aim of this work is to propose an efficient quasistatic contact approach to reduce the numerical stiffness of the system.

2 Global shape functions using the IGA

The floating frame of reference formulation is a well established approach when simulating flexible multibody systems [1]. As visualized in Fig. 1, large nonlinear rigid body motion of the body frame K_R can be described within the inertial frame K_I. Given that the elastic body deformations remain small and linear, they can be described conveniently in the body frame. Elastic deformations are approximated by global shape functions and their corresponding elastic coordinates $\mathbf{q}_e$, which in this work are determined with the IGA. The IGA uses basis-splines (B-splines) defined in the so-called parameter space, which can be seen in Fig. 2. The figure also shows the knots which span the elements. In order to visualize the geometry, the parameter space is

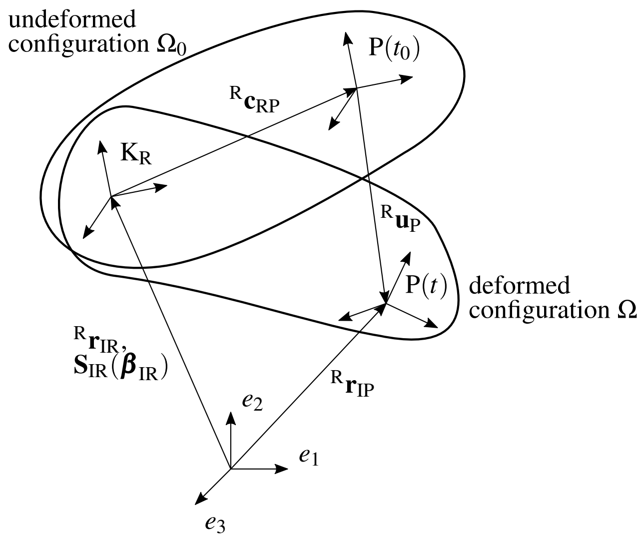

Figure 1: Floating frame of reference formulation

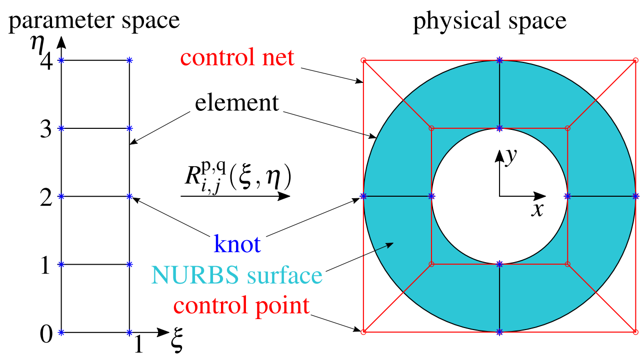

Figure 2: Parameter and physical space in the IGA

transformed into the physical space. This transformation is accomplished with the non-uniform rational B-splines (NURBS). The NURBS are the local shape functions of the isogeometric elements and enable the exact representation of the geometry. For a more detailed introduction to the IGA, see, for instance, [3].
To extract the global shape function, the full IGA model is reduced. To capture precise deformations and stresses in the area of contact, a large number of eigenmodes would be required. Instead, Craig-Bampton method is applied. It uses a combination of eigenmodes and static shape functions to describe the flexible body. Low frequency eigenmodes represent the global deformation, and high eigenfrequency static shape functions capture the local deformation in the contact area. The resulting equations of

motion

$$\underbrace{\begin{bmatrix} m\mathrm{E} & m\tilde{\mathbf{c}}^\mathsf{T} & \mathbf{C}_\mathrm{t}^{\mathrm{lf}\mathsf{T}} & \mathbf{C}_\mathrm{t}^{\mathrm{hf}\mathsf{T}} \\ m\tilde{\mathbf{c}} & \mathbf{I} & \mathbf{C}_\mathrm{r}^{\mathrm{lf}\mathsf{T}} & \mathbf{C}_\mathrm{r}^{\mathrm{hf}\mathsf{T}} \\ \mathbf{C}_\mathrm{t}^\mathrm{lf} & \mathbf{C}_\mathrm{r}^\mathrm{lf} & \overline{\mathbf{M}}_\mathrm{e}^\mathrm{lf} & \mathbf{0} \\ \mathbf{C}_\mathrm{t}^\mathrm{hf} & \mathbf{C}_\mathrm{r}^\mathrm{hf} & \mathbf{0} & \overline{\mathbf{M}}_\mathrm{e}^\mathrm{hf} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} {}^\mathrm{R}\dot{\mathbf{v}}_\mathrm{IR} \\ {}^\mathrm{R}\dot{\boldsymbol{\omega}}_\mathrm{IR} \\ \ddot{\mathbf{q}}_\mathrm{e}^\mathrm{lf} \\ \ddot{\mathbf{q}}_\mathrm{e}^\mathrm{hf} \end{bmatrix}}_{\dot{\mathbf{z}}_\mathrm{II}} = \underbrace{\begin{bmatrix} \mathbf{h}_\mathrm{dt} \\ \mathbf{h}_\mathrm{dr} \\ \mathbf{h}_\mathrm{de}^\mathrm{lf} \\ \mathbf{h}_\mathrm{de}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\mathrm{d}} + \underbrace{\begin{bmatrix} \mathbf{h}_\mathrm{bt} \\ \mathbf{h}_\mathrm{br} \\ \mathbf{h}_\mathrm{be}^\mathrm{lf} \\ \mathbf{h}_\mathrm{be}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\mathrm{b}} - \underbrace{\begin{bmatrix} \mathbf{h}_{\omega\mathrm{t}} \\ \mathbf{h}_{\omega\mathrm{r}} \\ \mathbf{h}_{\omega\mathrm{e}}^\mathrm{lf} \\ \mathbf{h}_{\omega\mathrm{e}}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\omega} - \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \overline{\mathbf{K}}_\mathrm{e}^\mathrm{lf}\mathbf{q}_\mathrm{e}^\mathrm{lf} \\ \overline{\mathbf{K}}_\mathrm{e}^\mathrm{hf}\mathbf{q}_\mathrm{e}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\mathrm{e}} \tag{1}$$

are separated in low frequency (lf) and high frequency (hf) modes. The wide frequency band of low and high frequency modes results in a numerically stiff system of equations. The next section addresses the challenge of numerical stiffness and proposes a solution.

3 Quasistatic contact model

Usually, an impact can be divided into three phases: the pre-contact phase, the contact phase and the post contact phase. In the pre-contact phase, the modes of the flexible multibody system are not yet excited. Therefore, the integrator can use larger step sizes. In case of contact, the whole frequency band of the equations of motion (1) is excited. Due to the separation into low and high frequency modes, the equations of motion (1) become numerically stiff requiring small step sizes. In the post-contact phase, the modes remain excited and the solution of the equations of motion (1) is still numerically expensive. Although, no highly dynamic contact forces occur in the post-contact phase, the step sizes are nevertheless smaller than in the pre-contact phase. If subsequent contacts are simulated, the computation until the next contact is expensive.
One solution is to critically damp the high frequency modes with modal damping. This is already achieved in [2]. This work proposes quasistatic contact as an alternative approach. It is shown in [4] that the high frequency modes only have small influence on the dynamics and can be neglected in the equations of motion (1). Neglecting the dynamics leads to updated equations of motion

$$\underbrace{\begin{bmatrix} m\mathrm{E} & m\tilde{\mathbf{c}}^\mathsf{T} & \mathbf{C}_\mathrm{t}^{\mathrm{lf}\mathsf{T}} & \mathbf{0} \\ m\tilde{\mathbf{c}} & \mathbf{I} & \mathbf{C}_\mathrm{r}^{\mathrm{lf}\mathsf{T}} & \mathbf{0} \\ \mathbf{C}_\mathrm{t}^\mathrm{lf} & \mathbf{C}_\mathrm{r}^\mathrm{lf} & \overline{\mathbf{M}}_\mathrm{e}^\mathrm{lf} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} {}^\mathrm{R}\dot{\mathbf{v}}_\mathrm{IR} \\ {}^\mathrm{R}\dot{\boldsymbol{\omega}}_\mathrm{IR} \\ \ddot{\mathbf{q}}_\mathrm{e}^\mathrm{lf} \\ \mathbf{0} \end{bmatrix}}_{\dot{\mathbf{z}}_\mathrm{II}} = \underbrace{\begin{bmatrix} \mathbf{h}_\mathrm{dt} \\ \mathbf{h}_\mathrm{dr} \\ \mathbf{h}_\mathrm{de}^\mathrm{lf} \\ \mathbf{h}_\mathrm{de}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\mathrm{d}} + \underbrace{\begin{bmatrix} \mathbf{h}_\mathrm{bt} \\ \mathbf{h}_\mathrm{br} \\ \mathbf{h}_\mathrm{be}^\mathrm{lf} \\ \mathbf{0} \end{bmatrix}}_{\mathbf{h}_\mathrm{b}} - \underbrace{\begin{bmatrix} \mathbf{h}_{\omega\mathrm{t}} \\ \mathbf{h}_{\omega\mathrm{r}} \\ \mathbf{h}_{\omega\mathrm{e}}^\mathrm{lf} \\ \mathbf{0} \end{bmatrix}}_{\mathbf{h}_\omega} - \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \overline{\mathbf{K}}_\mathrm{e}^\mathrm{lf}\mathbf{q}_\mathrm{e}^\mathrm{lf} \\ \overline{\mathbf{K}}_\mathrm{e}^\mathrm{hf}\mathbf{q}_\mathrm{e}^\mathrm{hf} \end{bmatrix}}_{\mathbf{h}_\mathrm{e}} \tag{2}$$

where the last equation is an algebraic equation. Only the contact forces $\mathbf{h}_\mathrm{de}^\mathrm{hf}$ and the inner forces $\overline{\mathbf{K}}_\mathrm{e}^\mathrm{hf}\mathbf{q}_\mathrm{e}^\mathrm{hf}$ remain. These components are essential to precisely represent the elastic deformations in the contact area. The algebraic equation

$$\mathbf{f}(\mathbf{q}_\mathrm{e}^\mathrm{hf}) = \mathbf{h}_\mathrm{de}^\mathrm{hf}(\mathbf{r}_\mathrm{IR}, \boldsymbol{\beta}_\mathrm{IR}, \mathbf{q}_\mathrm{e}^\mathrm{lf}, \mathbf{q}_\mathrm{e}^\mathrm{hf}) - \overline{\mathbf{K}}_\mathrm{e}^\mathrm{hf}\mathbf{q}_\mathrm{e}^\mathrm{hf} = \mathbf{0} \tag{3}$$

is removed from the differential algebraic equations (2) and solved in a separate quasistatic contact problem. The highly nonlinear quasistatic contact equation (3) needs to be solved for the high frequency elastic coordinates $\mathbf{q}_\mathrm{e}^\mathrm{hf}$ in every time step with Newton's method. In practice, solving Eq. (3) is numerically challenging. However, advantages of quasistatic contact are the reduced number of differential equations and numerical stiffness of the equations of motion. As a result, the contact and post-contact phase can be computed with larger step sizes.

4 Application example

As an application example, two double pendulums are simulated. Each double pendulum consists of a flexible rod and flexible sphere modeled with the IGA. Initially, one pendulum is deflected and the other pendulum is at rest. This simulation setup includes large rigid body motions in the pre- and post-contact phase. The application example is therefore well suited to compare a contact simulation based on Eq. (1) with modal damping to a quasistatic contact simulation. The focus of the analysis is on the required computation time and accuracy of the results. The analytical solution of Hertz [5] is used as a reference.

References

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