A beam-to-rigid body frictional contact formulation: application to yarn-mandrel interactions

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EXTENDED ABSTRACT

1 Introduction

Nonlinear beam theories are proven to be effective for the modelling of highly flexible engineering components undergoing large deformations. In conjunction with beam models, constraints of unilateral nature are introduced for the modelling of frictional contact interactions with rigid or flexible bodies. Using nonsmooth models (as opposed to compliant models), the unilateral constraint can be expressed as a Signorini condition together with a Coulomb friction law which result in jumps at velocity level. Due to discontinuities arising from impact events with friction, a special treatment is further required for the numerical time integration to solve the nonlinear system of equations with nonsmooth phenomena.

In this work, the frictional contact interactions between beams and rigid bodies based on a point-to-point approach shall be addressed. As a representative problem, the winding of a slender textile yarn around a rigid mandrel of compound geometry (convex and concave contours) shall be considered. If the contact is frictionless, the winding trajectory follows a geodesic path. Although this setting simplifies the computational effort and lets itself to analytical solutions for simple geometries, such geodesic solutions are inaccurate and/or impractical in various cases, especially during drastic changes in the mandrel geometry (sharp edges etc.). Therefore, non-geodesic yarn paths [1] are observed experimentally which deviate from the geodesics and are therefore unstable. Such deviations are caused by the lateral frictional force which stabilizes the yarn configuration and prevents slipping off along the mandrel surface. Particularly, a non-geodesic yarn trajectory on a mandrel can be obtained when the contact is in a sticking state. Additionally, for the simple case of a cylindrical mandrel, a mortar formulation for distributed contact over a continuous patch (line) as proposed in [2] shall be compared to the point-to-point approach. The cylindrical mandrel in this case shall be then modelled as a stocky beam and the contact with the yarn shall be modelled as a line contact.

2 Method

The slender textile yarn shall be modelled as a geometrically exact beam on the Lie group $SE(3)$ using the dynamic beam formulations proposed in [3]. The time discrete equations are obtained using the decoupled version of the nonsmooth generalized-α (NSGA) [4] which sequentially splits the integration of motion into three sub-problems for smooth motion and nonsmooth corrections at position and velocity levels. For example, at velocity level, the frictional contact problem in the tangential direction is expressed as

$$-(\mathbf{g}^j_{Tq,n+1}\mathbf{v}_{n+1} + e^j_T\mathbf{g}^j_{Tq,n}\mathbf{v}_n) \in \partial\psi_{C(\Lambda^j_{N,n+1})}(\mathbf{\Lambda}^j_{T,n+1}) \qquad \text{if } g^j_N(q_{n+1}) \leq 0, \tag{1}$$

In this expression, q is the configuration variable, $\mathbf{v}$ is the velocity, $\mathbf{g}^j$ is the relative position for the contact j with the normal component g^j_N and tangential component $\mathbf{g}_T$, and e^j_T is the tangential restitution coefficient which is chosen as $e^j_T = 0$ for contact involving flexible bodies. Λ^j_N and $\mathbf{\Lambda}^j_T$ are the normal and tangential components of the Lagrange multiplier respectively representing the impulse, $\mathbf{g}_{Tq}$ is the tangential constraint gradient and $\psi_{C(\Lambda_N)}$ is the indicator function of the section of the Coulomb friction cone.

As proposed in [5], the centerline of the yarn beam shall be represented using proxy collision geometries (spherical, cylindrical etc.). The mandrel surface shall be treated as the master plane of rigid body and the collision geometries (attached to the beam) as the slave. The frictional interactions between the bodies shall be computed from the augmented Lagrangian which includes both normal and tangential components. For the robust handling of a large number of frictional impacts, Newton-type solvers may suffer from ill-conditioning and singularity issues in the Jacobian. Therefore, a Gauß-Seidel based solver So-bogus [6] shall be exploited for solving the discrete equations.

3 Preliminary results

A slender beam of radius $r = 0.001$ m with length $l = 4$ m and material properties of basalt fibres ($E = 90$ GPa, $\nu = 0.21$) has been used for winding around a cylindrical rigid body of 0.4 m radius. The beam is discretized using 50 finite elements and spherical collision elements (of radius = r) are attached to the nodes of the beam. The yarn-mandrel interactions are therefore solved using a nonsmooth rigid body contact formulation with friction using Odin software [7]. The total simulation time is 12 seconds, with time step size $h = 0.005$ seconds. A preliminary investigation of the yarn-mandrel interactions has been performed

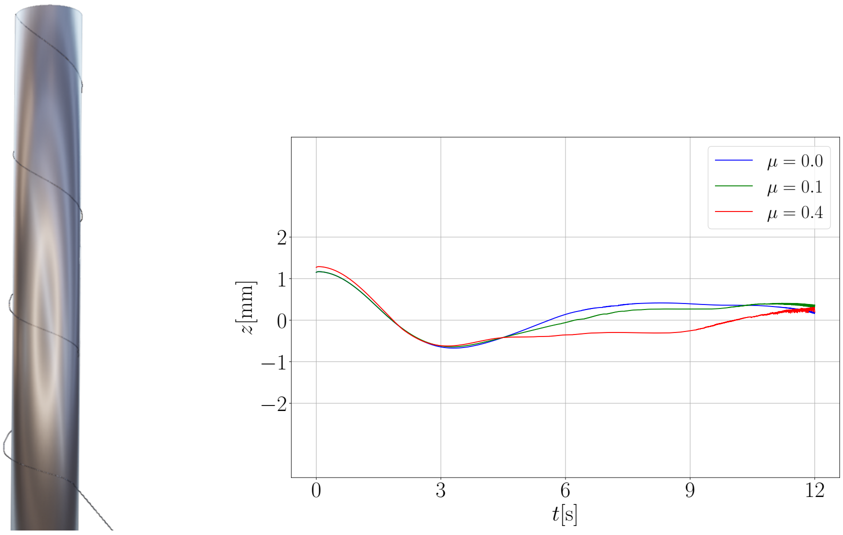

Figure 1: Frictional dynamic yarn-mandrel interactions using beam-to-rigid body contact. Time evolution of position of the center node of the yarn along the longitudinal axis (z) of the mandrel for different friction coefficients.

in Figure 1 for one frictionless and two frictional cases ($\mu = 0.1$ and 0.4). The time evolution of position of the center node of the beam (node 25) has been selected for the study. As expected in winding using a higher value of the friction coefficient ($\mu = 0.4$), the sliding motion of the node is significantly postponed as compared to $\mu = 0.1$ and 0.0.

4 Conclusion

A beam-to-rigid body contact formulation with friction based on a point-to-point approach has been proposed to study the dynamics of yarn deposition on compound mandrel shapes. As a preliminary study, a slender beam represented by discrete collision geometries in contact with a cylindrical mandrel has been presented. A robust handling of a large number of rigid contacts using a Gauß-Seidel solver can be observed. In the future, the stability of yarn paths shall be further studied for abrupt changes in mandrel geometry and comparisons with yarn paths from geometric models in literature shall also be presented.

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