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Flexible Mounting Elements for Cosserat Rods

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EXTENDED ABSTRACT

1 Introduction

Nowadays, the simulation of flexible cables and hoses is an important aspect in early development phases, especially in vehicle engineering. Due to the slenderness of cables and hoses, geometrically exact rod models as presented in [1] are suitable to achieve fast and accurate simulations, even for dynamic simulations (see [2]). Besides cables and hoses, also many of their mounting elements show significant flexibility. For realistic simulation results, this flexibility must be taken into account. To this end, we combine our flexible rod model with flexible clip models. Typically, for acceptable simulation performance, those clip models are derived from FE models by common reduction techniques (e.g. substructuring [3]).

2 Methodology

We consider a Cosserat rod with a staggered grid discretization, where translatory degrees of freedom live in the nodes and rotatory degrees of freedom live on the edges. Thus a rod consisting of N_s segments is defined by N_s+1 nodes $\mathbf{x}_n \in \mathbb{R}^3, n \in \{0,\dots,N_s\}$ and N_s edge quaternions $\mathbf{p}_\nu \in \mathbb{H} \cong \mathbb{R}^4, \nu \in \{\frac{1}{2},\dots,N_s-\frac{1}{2}\}$. For clamped rod ends, we additionally introduce boundary quaternions $\mathbf{p}_0$ resp. $\mathbf{p}_{N_s}$, such that, in total, we end up with $N = 3(N_s+1)+4(N_s+2)$ degrees of freedom. The state of the rod is given by $\mathbf{q} = (\mathbf{p}_0, \mathbf{x}_0, \mathbf{p}_{1/2}, \mathbf{x}_1, \dots, \mathbf{p}_{N_s-1/2}, \mathbf{x}_{N_s}, \mathbf{p}_{N_s}) \in \mathbb{R}^N$. In order to describe rotations, quaternions $\mathbf{p}_I$ with $I \in \{0,\frac{1}{2},\dots,N_s-\frac{1}{2},N_s\}$ must be unit quaternions $\mathbf{p}_I \in \mathsf{S}^3 := \{\mathbf{p} \in \mathbb{R}^4 \| \|\mathbf{p}\| = 1\}$. Therefore we get $M = N_s+2$ constraints, each given by $g_I(\mathbf{q}) = \frac{1}{2}(\mathbf{p}_I \cdot \mathbf{p}_I - 1)$. The potential energy of the discrete Cosserat rod is given by the nonlinear scalar function $E : \mathbb{R}^N \longrightarrow \mathbb{R}, \mathbf{q} \longmapsto E(\mathbf{q})$, which consists of the bending, torsion, tension, shear and gravitational energy (see [1, 2] for details).

The flexible clip is assumed to have interface nodes $\mathbf{u} \in \mathbb{R}^L$, given in local clip coordinates. We assume that there exists a potential $P : \mathbb{R}^L \longrightarrow \mathbb{R}, \mathbf{u} \longmapsto P(\mathbf{u})$, which describes the potential energy of the clip as a function of these interface nodes $\mathbf{u}$. Additionally we assume that there exists a function $\psi : \mathbb{R}^L \longrightarrow \mathbb{R}^L, \mathbf{u} \longmapsto \mathbf{Y}$ mapping the local interface nodes to world coordinates. To relate the clip's interface nodes to the discrete rod, we introduce the function $\varphi : \mathbb{R}^L \times \mathbb{R}^N \longrightarrow \mathbb{R}^L, (\mathbf{Y}, \mathbf{q}) \longmapsto \mathbf{X}$, which maps the global interface nodes $\mathbf{Y} \in \mathbb{R}^L$ to local coordinates $\mathbf{X} \in \mathbb{R}^L$. It has to be noted that this mapping also depends on the configuration $\mathbf{q} \in \mathbb{R}^N$ of the discrete rod. Additionally we introduce the function $\phi : R^L \times \mathbb{R}^N \longrightarrow \mathbb{R}^L, (\mathbf{X}, \mathbf{q}) \longmapsto \mathbf{Y}$ which is the inverse of φ with respect to the first variable and, therefore, for all $\mathbf{q} \in \mathbb{R}^N$ and $\mathbf{Y} \in \mathbb{R}^L$ it holds $\mathbf{Y} = \phi\left(\varphi\left(\mathbf{Y}, \mathbf{q}\right), \mathbf{q}\right)$. In Figure 1 the different mappings are visualized.

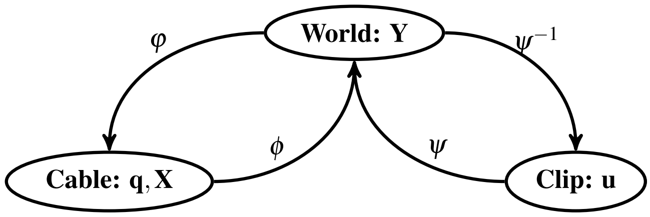

Figure 1: Diagram illustrating the mappings between cable, world and clip coordinates.

We relate the clip and the rod in an arbitrary but fixed initial configuration $\mathbf{u}_0$ and $\mathbf{q}_0$ and compute the corresponding local coordinates $\mathbf{X}_0 := \varphi(\psi(\mathbf{u}_0), \mathbf{q}_0)$ which subsequently are kept constant. This leads to the nonlinear optimization problem for rod state $\mathbf{q}$ and the local clip interface nodes $\mathbf{u}$ given by

$$\min_{\mathbf{q}\in\mathbb{R}^N \mathbf{u}\in\mathbb{R}^L} E(\mathbf{q}) + P(\mathbf{u}) \tag{1}$$

$$\text{s.t} \qquad \mathbf{g}(\mathbf{q}) = \mathbf{0} \tag{2}$$

$$\phi(\mathbf{X}_0, \mathbf{q}) - \Psi(\mathbf{u}) = \mathbf{0} \tag{3}$$

By applying Ψ^{-1} to (3) we get $\mathbf{u} = \Psi^{-1}(\phi(\mathbf{X}_0, \mathbf{q}))$ and, thus, can explicitly write $\mathbf{u}$ in terms of $\mathbf{q}$. Inserting this in the potential energy of the clip, we arrive at $\tilde{P}(\mathbf{q}) := P(\Psi^{-1}(\phi(\mathbf{X}_0, \mathbf{q})))$. Hence the optimization problem only depends on the variable $\mathbf{q}$.

3 Flexible Clips for Cables

As application case we consider flexible clips for cables and hoses. The clip model is given as reduced substructure model, generated by static condensation, only retaining the relevant interface nodes. Let $\mathbf{u} = (\mathbf{u}_1^T, \ldots, \mathbf{u}_{L_C}^T)^T$ be the local displacements of the interface nodes, i.e. the degrees of freedom of the clip, and let K by the stiffness matrix of the clip. Hence the potential energy of the clip is given by $P(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T \mathsf{K}\mathbf{u}$. Further, let $\mathbf{z} = (\mathbf{z}_1^T, \ldots, \mathbf{z}_{L_C}^T)^T$ be the local position of the interface nodes. Together with the global clip position $\mathbf{y}_C$ and rotation R_C, the transformation to world coordinates is given by $\Psi_I(\mathbf{u}_I) = \mathbf{y}_C + \mathsf{R}_C\,(\mathbf{z}_I + \mathbf{u}_I) = \mathbf{Y}_I$, for $I \in 1, \ldots, L_C$.
Now, also for the cable we need to define the transformation from world to local coordinates and vice versa. In the case of rod models it seems obvious to use cylinder-type coordinates, such that for each $\mathbf{Y}_I \in \mathbb{R}^3$ we define $\mathbf{X}_I = (s_I, d_I, \alpha_I)^T = \varphi(\mathbf{Y}_I, \mathbf{q})$, where s_I is the corresponding arc length, d_I is the distance to the centreline and α_I is the angular position in the local cable cross section (specified by the edge quaternion). The back transformation from local to world coordinates $\mathbf{Y}_I = \phi_I(\mathbf{X}_I, \mathbf{q})$ is defined correspondingly.

Figure 2 shows an example of a simple clip geometry. On the left one can see the full FE model of the clip and the definition of the interface nodes (in red), which will be retained also for the reduced model. On the right one can see a flexible cable, which is mounted at the reduced clip model.

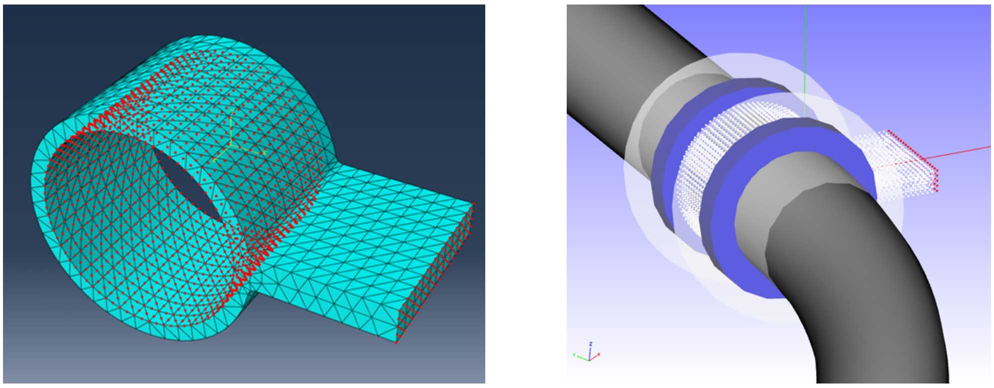

Figure 2: **Left:** FE-model of the clip with red interface nodes. **Right:** Flexible cable model mounted to the reduced clip model.

4 Conclusion and outlook

The above stated methodology, which utilizes known techniques to couple flexible components into multibody systems [4, 5], provides a versatile and general approach to couple our flexible cable model with additional flexible components. This generality is important, since mounting elements like clips for cables and hoses show various geometries and, especially, have very different support ranges (i.e. surface area in contact with the flexible rod). While the above optimization problem only solves static (or quasi-static) problems, we also plan to extend the methodology to dynamic problems.

References

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