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Modeling of the Achilles Subtendons in a Framework of the Absolute Nodal Coordinate Formulation

Leonid Obrezkov[1], Taija Finni[2], Marko K. Matikainen[1]

[1] Mechanical Engineering
LUT University
Yliopistonkatu 34, Lappeenranta, 53850, Finland
leonid.obrezkov@lut.fi, marko.matikainen@lut.fi

[2] Faculty of Sport and Health Sciences
University of Jyväskylä
Rautpohjankatu 8 Jyväskylä, 40014, Finland
taija.m.juutinen@jyu.fi

EXTENDED ABSTRACT

1 Introduction

Computational efficiency is crucial for the multibody system dynamics analysis [1], especially in the case of sophisticated systems such as human body parts. The absolute nodal coordinate formulation (ANCF) is a nonlinear finite element approach proposed for the large deformation dynamics analysis [2], which has shown its effectiveness [3]. The study aims to present the Achilles tendon as a beam-like structure described within this formulation. The beams are in a framework of the ANCF using the three-nodded higher-order continuum-based ANCF beam element denoted as 3363 in [1]. Such as tendons consisting of three sub-tendons with arbitrary cross sections, Green's integral formula for continuum-based ANCF beam elements is applied [4] to capture their cross-section forms. The contact is described by the method presented in [5]. Then, the human Achilles tendon, as one example, is considered as a multi-beam structure [6]. The example additionally demonstrates the potential of the cross-section description because the previous FE-based models of the tendon require a fine element mesh [7].

2 Numerical integration scheme via Green's integral formula

Here, the integration procedure introduced in [4] and applied in [6] is shortly revised. Let us consider a closed domain Ω, which has a piece-wise border $\partial\Omega$ with points V_i on it. Besides, the lines $[V_i, V_{i+1}]$ have several additional points, such $P_{i1} = V_i, P_{i2}, ..., P_{im_i} = V_{i+1}$. a parametrization is recalled:

$$[\alpha_{ij}^{\xi}, \beta_{ij}^{\xi}] = \left[0, \sum_{j=1}^{m_i-1} \Delta t_{ij}\right], \mid \Delta t_{ij} \mid = \mid P_{ij+1}^{\xi} - P_{ij}^{\xi} \mid, \ j = 1, ..., m_i - 1. \tag{1}$$

Then each line $[V_i, V_{i+1}]$ is tracked by a spline curve $S_i(t) = (S_{i1}(t), S_{i2}(t))$ the degree of p_i, where $p_i \leq m_i - 1$. The cubature formula with the $2n-1$ polynomial exactness degree over Ω domain then has the form as follows.

$$I_{2n-1} = \sum_{\lambda \in \Lambda_{2n-1}} \mathrm{w}_{\lambda} f(\eta_{\lambda}, \zeta_{\lambda}), \tag{2}$$

where

$$\Lambda_{2n-1} = \{\lambda = (i, j, k, h) : 1 \leqslant i \leqslant \varphi, 1 \leqslant j \leqslant m_i - 1, \ 1 \leqslant k \leqslant n_i, 1 \leqslant h \leqslant n\}, \tag{3}$$

and w_{λ}, η_{λ} and ζ_{λ} are:

$$\eta_{\lambda} = \frac{S_{i1}(q_{ijk}) - \Xi}{2} \tau_h^n + \frac{S_{i1}(q_{ijk}) + \Xi}{2}, \ \zeta_{\lambda} = S_{i2}(q_{ijk}), \tag{4}$$

$$w_{\lambda} = \frac{\Delta t_{ij}}{4} \omega_k^{n_i} \omega_h^n (S_{i1}(q_{ijk}) - \Xi) \frac{\mathrm{d}S_{i2}(t)}{\mathrm{d}t} \mid_{t=q_{ijk}}, \ q_{ijk} = \frac{\Delta t_{ij}}{2} \tau_k^{n_i} + \frac{t_{ij+1} + t_{ij}}{2}, \ \Delta t_{ij} = t_{ij+1} - t_{ij}, \tag{5}$$

$$n_i = \begin{cases} np_i + p_i/2, \ p_i \text{ is even}, \\ np_i + (p_i+1)/2, \ p_i \text{ is odd}. \end{cases} \tag{6}$$

Thus, only $\tau_k^{n_i}$, $\omega_k^{n_i}$ and Ξ are to be defined. Ξ is an arbitrary straight line. $\tau_k^{n_i}$, $\omega_k^{n_i}$ are the nodes and weights, respectively, of the Gauss–Legendre quadrature formula of the exactness degree $2n_i - 1$ on $[-1, 1]$. The presented approach is used to obtain the cross section of the sub-tendons shown in Fig 1a-1c.

3 Contact Formulation

The using surface-to-surface contact formulation presented in [5] is recalled to combine tree sub-tendons given as flexible deformable beams with arbitrary cross sections. The following body can be formed (see Fig. 2).

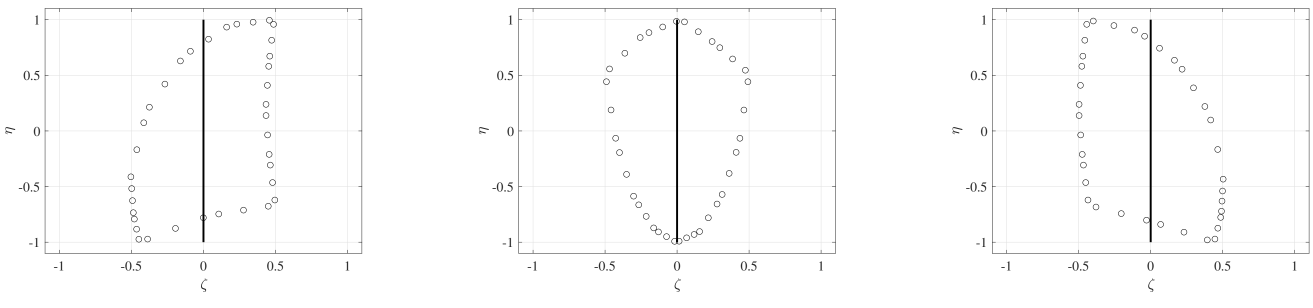

Figure 1: Integration approximations of the sub-tendons by the Gauss–Green cubature formula.

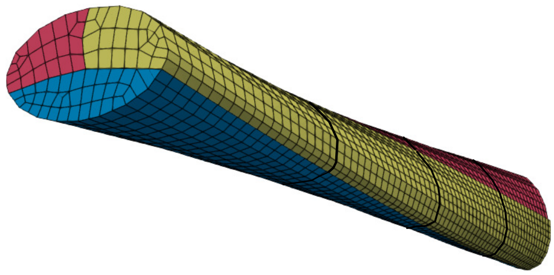

Figure 2: Tendon beam-based approximation.

4 Conclusion

This work considers the beam-like structures with non-standard cross-sections for the human Achilles tendon description. The convergence of the system based on the ANCF element was checked with varying different meshes [6].

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