# Beam Modeling in a Floating Frame of Reference for Torsion Dynamics of Helicopter Rotor Blades

Felix Weiss, Joshua Merlis

German Aerospace Center (DLR)
Institute of Flight Systems
Lilienthalplatz 7, 38108 Braunschweig, Germany
felix.weiss@dlr.de

## EXTENDED ABSTRACT

Helicopter rotor blades are usually slender structures with a rotor radius much larger than the chord length and thickness. This geometrical property encourages modeling the blade as a one dimensional (1D) beam, which saves computational costs compared to a 3D structural model. For this reason, multi-physics tools for helicopter (rotor) simulations — so-called comprehensive codes — commonly feature beam models to simulate the blades. In the ongoing development of DLR's Versatile Aeromechanics Simulation Tool (VAST) [1], an elastic beam model is integrated into the Multibody System (MBS) based on the Floating Frame of Reference (FFR) formulation [2]. Although the application of this formulation to 1D beam models has already been addressed in the literature, e.g. [3], the challenge remains to properly model the torsion dynamics of rotor blades — especially under high centrifugal loads. To do so, this work suggests the consideration of rotational shape functions in the inertia shape integrals and in the application of gravitational, inertial, and external loads. This modified approach is validated based on the structural analysis of a rotor blade with complex geometrical properties.

The nodal position states $\mathbf{r}_\mathrm{I}$ and velocity states $\mathbf{r}_\mathrm{II} = \dot{\mathbf{r}}_\mathrm{I}$ of the Finite Element (FE) model describe the body deformation with respect to the FFR, which moves ("floats") relative to the inertial frame by the linear and angular velocities $\mathbf{v}$ and $\omega$. These vectors represent the rigid body portion of motion with the corresponding mass matrix $\mathbf{m}_\mathrm{RR}$, inertia tensor $\bar{\mathbf{J}}_{\theta\theta}$, and coupling matrix $\tilde{\bar{\mathbf{S}}}_\mathrm{t}^\mathrm{T}$, see the upper part of equation 1:

$$\begin{bmatrix} \mathbf{m}_\mathrm{RR} & \tilde{\bar{\mathbf{S}}}_\mathrm{t}^\mathrm{T} & \mathbf{0} & \bar{\mathbf{S}} \\ & \bar{\mathbf{J}}_{\theta\theta} & \mathbf{0} & \bar{\mathbf{J}}_{\theta\mathrm{f}} \\ & & \mathbf{I} & \mathbf{0} \\ sym. & & & \mathbf{M}_\mathrm{ff} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\omega} \\ \dot{\mathbf{r}}_\mathrm{I} \\ \dot{\mathbf{r}}_\mathrm{II} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & -\mathbf{K}_\mathrm{ff} & -\mathbf{D}_\mathrm{ff} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \omega \\ \mathbf{r}_\mathrm{I} \\ \mathbf{r}_\mathrm{II} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_\mathrm{g}^\mathrm{R} \\ \mathbf{Q}_\mathrm{g}^\theta \\ \mathbf{0} \\ \mathbf{Q}_\mathrm{g}^\mathrm{f} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_\mathrm{v}^\mathrm{R} \\ \mathbf{Q}_\mathrm{v}^\theta \\ \mathbf{0} \\ \mathbf{Q}_\mathrm{v}^\mathrm{f} \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_\mathrm{e}^\mathrm{R} \\ \mathbf{Q}_\mathrm{e}^\theta \\ \mathbf{0} \\ \mathbf{Q}_\mathrm{e}^\mathrm{f} \end{bmatrix} \tag{1}$$

The second order differential equation of the FE system has been converted to first order form (lower part of equation 1), in which the FE system's mass $\mathbf{M}_\mathrm{ff}$, stiffness $\mathbf{K}_\mathrm{ff}$, and damping $\mathbf{D}_\mathrm{ff}$ matrices are found. $\mathbf{K}_\mathrm{ff}$ is composed of both the structural and geometric stiffness. The rightmost entries $\bar{\mathbf{S}}$ and $\bar{\mathbf{J}}_{\theta\mathrm{f}}$ in the upper part of the overall mass matrix constitute coupling terms between the rigid body motion and the flexible motion. The right hand side includes gravitational loads $\mathbf{Q}_\mathrm{g}$, inertial loads $\mathbf{Q}_\mathrm{v}$, and external loads $\mathbf{Q}_\mathrm{e}$ acting on the rigid translatory, rigid rotational, and flexible motion (superscripts R, $\theta$, and f, respectively).

According to the general theory [2], several submatrices and vectors of equation 1 include volume integrals with both the mass density $\rho(x,y,z)$ and the translatory shape function matrix $\mathbf{S}_\mathrm{tra}(x,y,z)$ as integrands—the so-called inertia shape integrals. $x$, $y$, and $z$ are the coodinates of the volume increment with respect to the FFR. The shape function is also used to evaluate the kinematics of the system. For example, a particle's velocity relative to the FFR reads $\dot{\bar{\mathbf{u}}} = \mathbf{S}_\mathrm{tra}(x,y,z)\,\mathbf{r}_\mathrm{II}$ in the general 3D formulation. In contrast, for a 1D beam with torsional degrees of freedom as shown in Figure 1, $\mathbf{S}_\mathrm{tra}(\zeta)$ is insufficient to evaluate the kinematics of a particle that is located at a distance from the reference axis. $\zeta$ is the coordinate along this axis, and $\mathbf{u}_\mathrm{A}$ is the particle's offset

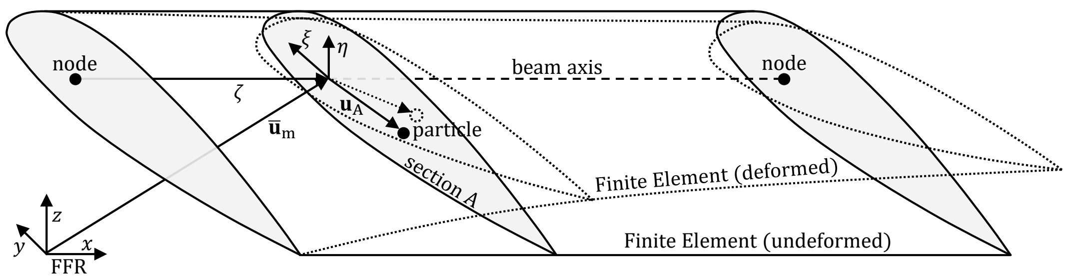


Figure 1: Particle in a cross section A moving due to torsional deformation of a 1D beam

from the reference axis within the cross section. The particle's velocity is $\dot{\bar{\mathbf{u}}} = \mathbf{S}_\mathrm{tra}(\zeta)\,\mathbf{r}_\mathrm{II} + \omega_\mathrm{flex} \times \mathbf{u}_\mathrm{A}$ with $\omega_\mathrm{flex} = \mathbf{S}_\mathrm{rot}(\zeta)\,\mathbf{r}_\mathrm{II}$. Here, the rotational shape function matrix $\mathbf{S}_\mathrm{rot}$ has been introduced, as presented in [4]. For a consistent consideration of $\mathbf{S}_\mathrm{rot}$ in equation 1, this paper proposes the usage of the complemented shape function matrix

$$\mathbf{S}(\zeta,\xi,\eta) = \mathbf{S}_\mathrm{tra}(\zeta) - \tilde{\mathbf{u}}_\mathrm{A}(\xi,\eta)\,\mathbf{S}_\mathrm{rot}(\zeta), \tag{2}$$

where $(\xi, \eta)$ are the coordinates within the cross section and the tilde symbol denotes the cross product operator. The expression $\dot{\mathbf{u}} = \mathbf{S}(\zeta, \xi, \eta)\,\mathbf{r}_{\mathrm{II}}$ evaluates to the same terms as given above. $\mathbf{S}$ is not only used in $\mathbf{M}_{\mathrm{ff}}$, $\bar{\mathbf{J}}_{\theta\mathrm{f}}$, and $\bar{\mathbf{S}}$, but also to calculate the inertial loads acting on flexible motion ($\mathbf{a}$ = FFR acceleration due to angular velocities in the MBS, $\bar{\mathbf{u}}$ = particle's position with respect to the FFR)

$$\mathbf{Q}_{\mathrm{v}}^{\mathrm{f}} = -\int_V \rho \mathbf{S}^{\mathrm{T}} \left(\mathbf{a} + \tilde{\omega}\tilde{\omega}\bar{\mathbf{u}} + 2\tilde{\omega}\dot{\bar{\mathbf{u}}}\right) \mathrm{d}V. \tag{3}$$

The influence of using the complemented shape function matrix $\mathbf{S} = \mathbf{S}_{\mathrm{tra}} - \tilde{\mathbf{u}}_{\mathrm{A}}\,\mathbf{S}_{\mathrm{rot}}$ (equation 2) instead of the original shape function matrix $\mathbf{S} = \mathbf{S}_{\mathrm{tra}}$ for calculating $\mathbf{Q}_{\mathrm{v}}^{\mathrm{f}}$ is demonstrated in the Campbell diagram of a double-swept prototype blade [5] of a helicopter rotor research program at Airbus, see Figure 2. At a normalized rotor speed of 1.0, the first normalized torsion/flap eigenfrequency in the experiment is 4.25, see the ●-marker. The VAST prediction with $\mathbf{S} = \mathbf{S}_{\mathrm{tra}}$ is 4.11, which is 3 % lower than in the experiment. In contrast, the result using $\mathbf{S}$ according to equation 2 is 4.20, which matches the experimental reference with an error of $-1$ % and also fits the numerical reference better. For the second torsion/flap mode (experimental normalized eigenfrequency = 5.50), the prediction error improves from $-5$ % to $+1$ %. The improvements are even more significant at higher rotor speeds. Phenomenologically, the change in torsion eigenfrequency is related to the so-called propeller moment, which constitutes one of the effects of centrifugal loads on the pitch motion and the torsional deformation of the rotor blade.

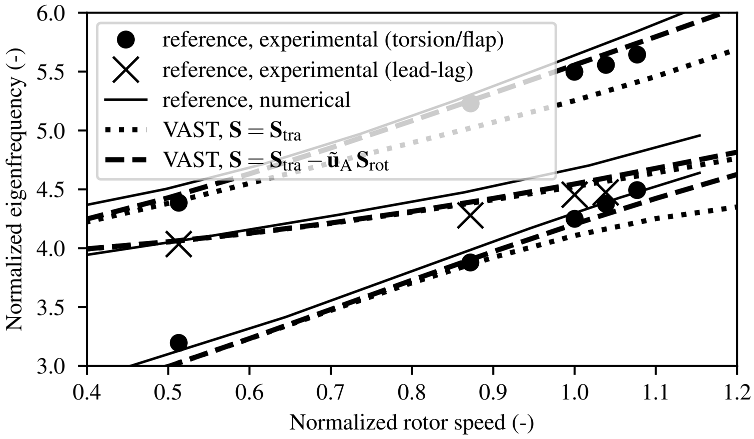


Figure 2: Campbell diagram (only selected modes) of a double-swept rotor blade, modeled with the original and complemented shape function matrices $\mathbf{S}$ for the calculation of $\mathbf{Q}_{\mathrm{v}}^{\mathrm{f}}$, reference results taken from [5]

## Conclusion

The FFR formulation is used to model helicopter rotor blades as 1D beams within an MBS. To appropriately account for the torsion dynamics of a rotor blade in this approach, rotational shape functions must be considered in the inertia shape integrals and when projecting loads (e.g. centrifugal forces) onto the flexible degrees of freedom. This can be achieved by using the complemented shape function matrix, which includes the effect of rotational deflection in combination with the beam axis offset of a particle. The added value of this modified FFR formulation is demonstrated based on the structural analysis of a double-swept rotor blade. The prediction of the torsion eigenfrequencies significantly improves when using the complemented shape function matrix in the calculation of inertial loads acting on the flexible degrees of freedom.

## References


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