Local Isotropic Compliance in Planar Parallel 3RRR Manipulators

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EXTENDED ABSTRACT

1 Introduction

Planar parallel 3RRR manipulators guarantee all the advantages of parallel manipulators but, at the same time, present their typical drawbacks [1, 2]. To improve their performance, many investigations applied different control techniques, such as proportional-derivative plus gravity compensation or computed torque control scheme [3]. In recent investigations, a proportional controller was implemented to achieve a particular response of the end-effector of serial manipulators. In kinetostatic conditions, this response is determined by the *local* isotropic compliance condition in the Special Euclidean Group SE(3), verified when the force vector is parallel to the displacement vector and, at the same time, the torque axis is parallel to the rotation axis. In this investigation, the isotropic compliance property is analyzed in the Special Euclidean Group SE(2) by considering the 3RRR planar parallel manipulator. A proper control strategy is defined and multibody simulations are performed to verify the effectiveness of the control system.

2 Formulation of the isotropic compliance problem in SE(3)

With reference to Fig. 1, point P represents the interaction point between the end-effector and the external environment.

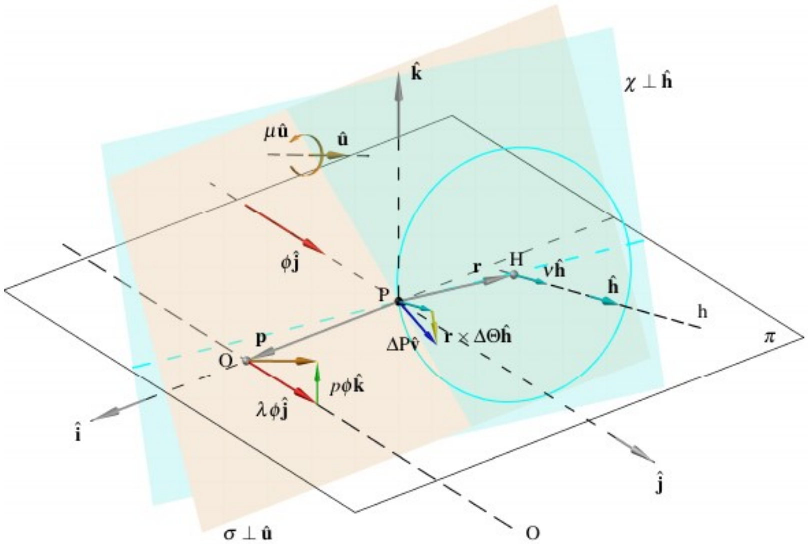

Figure 1: Wrench $\mathbf{w}$ and consequent twist $\mathbf{t}$ in the general case.

In general kinetostatic conditions, the manipulator posture and its dynamic characteristics determine the final pose of the end-effector. As a consequence, there are no specific relations between the unit vectors $\hat{\mathbf{j}}$ and $\hat{\mathbf{v}}$, that represent the directions of the applied force and the end-effector displacement, respectively. Analogously, there are no particular relations between the direction of the unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{h}}$, that represent the torque axis and the end-effector rotation axis, respectively.

The *local* isotropic compliance is condition of parallelism between the unit vectors $\hat{\mathbf{j}}$ and $\hat{\mathbf{v}}$, and between the unit vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{h}}$, and can be formulates as

$$\Delta \mathrm{P}\hat{\mathbf{v}} = c_d \phi \hat{\mathbf{j}} \qquad \text{and} \qquad \Delta\Theta\hat{\mathbf{h}} = c_r \mu \hat{\mathbf{u}} \,, \tag{1}$$

respectively, where c_d and c_r are scalars. In matrix form, eq. (1) can be rearranged as $\Delta\mathbf{S} = \mathbf{C}\mathbf{w}$, with

$$\mathbf{C} = \begin{bmatrix} c_d\mathbf{I} & 0 \\ 0 & c_r\mathbf{I} \end{bmatrix} . \tag{2}$$

On the other hand, the compliance matrix of a robotic manipulator is defined in the task space as $\mathbf{C} = \mathbf{J}\mathbf{k}^{-1}\mathbf{J}^{\mathrm{T}}$.

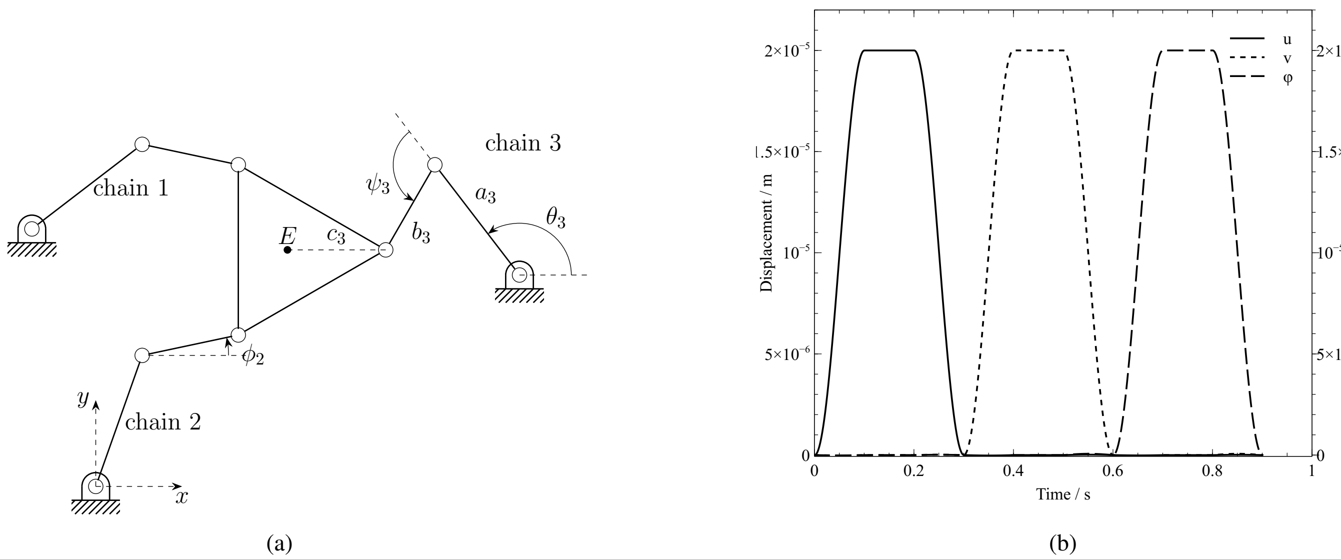

Figure 2: Nomenclature (a) and end-effector displacement and rotation for the controlled system (b).

In case of active stiffness regulation acting in parallel to the passive stiffness of the joints, the stiffness matrix in the joint space can be calculated as $\mathbf{k} = \mathbf{k}_\mathrm{p} + \mathbf{k}_\mathrm{c}$ [4]. Therefore, the control matrix $\mathbf{k}_\mathrm{c}$ can be exploited to transform the compliance matrix of the manipulator in the scalar-like matrix in eq. (2) that guarantees the achievement of the local isotropic compliance property. By resorting to the spectral decomposition, the passive stiffness matrix in the task space can be written as

$$\mathbf{K}_\mathrm{p} = \begin{bmatrix} \mathbf{U}_\mathrm{E}\mathbf{\Sigma}_\mathrm{E}{\mathbf{U}_\mathrm{E}}^\mathrm{T} & \mathbf{G} \\ \mathbf{G}^\mathrm{T} & \mathbf{U}_\mathrm{H}\mathbf{\Sigma}_\mathrm{H}{\mathbf{U}_\mathrm{H}}^\mathrm{T} \end{bmatrix}, \tag{3}$$

where the columns of $\mathbf{U}_\mathrm{E}$, $\mathbf{U}_\mathrm{H}$ form a set of orthonormal eigenvectors of $\mathbf{E}$ and $\mathbf{H}$, respectively, and $\mathbf{\Sigma}_\mathrm{E}$, $\mathbf{\Sigma}_\mathrm{H}$ are diagonal matrices with the eigenvalues of $\mathbf{E}$ and $\mathbf{H}$, respectively. Therefore, by considering eq. (3) and

$$\mathbf{K}_\mathrm{c} = \begin{bmatrix} \Delta\mathbf{E} & -\mathbf{G} \\ -\mathbf{G}^\mathrm{T} & \Delta\mathbf{H} \end{bmatrix}, \tag{4}$$

the overall stiffness matrix in the task space becomes a scalar-like matrix.

3 Results for the 3RRR planar parallel manipulator

The matrix $\mathbf{K}_\mathrm{c}$ has been calculated for the manipulator depicted in Fig. 2(a) and the effectiveness of the control system has been verified by MBDyn, a free general-purpose multibody solver developed at Politecnico di Milano (`http://mbdyn.org/`, [5, 6]).

The results presented in Fig. 2(b) show the end-effector loads, the corresponding displacement and the finite rotation vectors, u and Θ, computed with the control action. It is clear that when the feedback action is present, the solution provides almost perfectly the sought local isotropic compliance.

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