Adjoint-based feedforward control of two-degree-of-freedom planar robot

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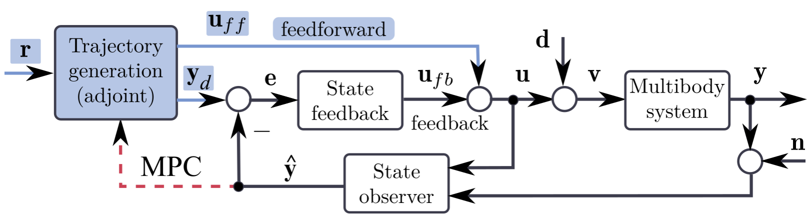
EXTENDED ABSTRACT

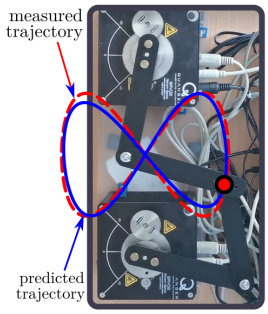
1 Background and contribution

Many recent state-of-the-art industrial and academic projects rely heavily on methods for interactive, reliable, safe, and accurate simulation of dynamic systems and related interdisciplinary phenomena. In order to exploit available information to the full extent, model-based methodologies have received much attention since they can be naturally integrated into a control loop as a feedforward (trajectory optimization, abbr. TO) component [1].

The indirect optimal control methodology provides offline (potentially online) trajectory generation tools that can be utilized as a feedforward signal in the control loop. The adjoint method, derived from this theoretical background, is a model-based approach emerging in multibody systems' (MBS) dynamics [2, 3]. This paper presents a practical implementation of the adjoint method in a feedback-feedforward control architecture shown in fig. 1a. We derive a mathematical model of an electromechanical device composed of a five-bar multibody system and two DC motors with a gear transmission (fig. 1b). Based on the derived outcome, we synthesize the input control signal and corresponding trajectory predicted by the model. Subsequently, these signals are introduced as reference values to the hardware and compared with classical control algorithms.



(a)



(b)

Figure 1: (a) A feedback–feedforward control architecture and (b) two-degree-of-freedom, closed-loop robotic MBS

2 Adjoint-based trajectory generation

The block diagram in fig. 1a presents a model-based control architecture that will be investigated herein. Symbol $\mathbf{r}$ denotes a reference signal that nominally must be enforced. Accordingly, this signal plays the role of an input to the adjoint-based optimization procedure founded on the mathematical model of the MBS. The optimization algorithm yields a control signal $\mathbf{u}_{ff}$ (theoretically) capable of carrying out the required maneuver. The response generated by the model for $\mathbf{u}_{ff}$ is depicted as $\mathbf{y}_d$, becoming the actual reference trajectory for the system. Due to the discrepancy between the model and the plant, as well as the presence of disturbances $\mathbf{d}$ and measurement noise $\mathbf{n}$, it is required to introduce a feedback loop that generates minor corrections $\mathbf{u}_{fb}$ during the execution on the hardware.

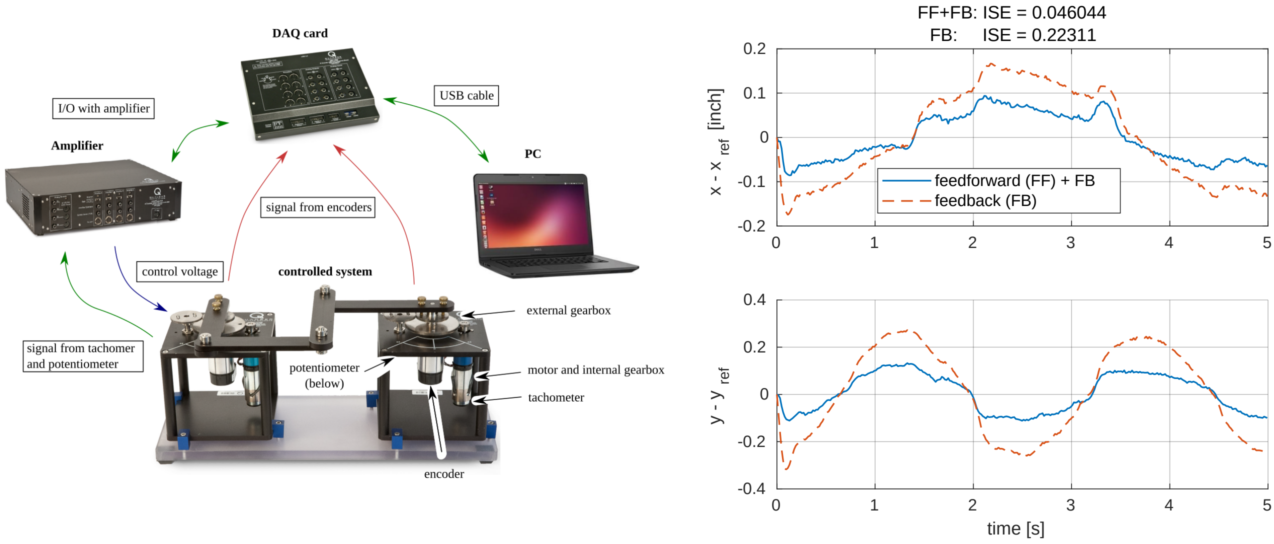
Solving for the trajectory in real-time can allow for its reevaluation as the simulation or task evolves. Repetitive trajectory generation can provide a feedback action that could itself be used for control purposes. [4]. This approach is the driving force behind model predictive control (MPC), which has been applied to many application domains over the last decades [5]. In this work, nevertheless, the TO component is computed offline, i.e. before execution on the hardware.

Given a prescribed trajectory $\mathbf{r}_\mathbf{P}^*(t)$ of the end-effector $\mathbf{P}$, we can utilize a mathematical model of the MBS in the form $\mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\mathbf{b}) = \mathbf{0}$ to solve the following fixed-time minimization problem:

$$J = \int_0^{t_f} \Big((\mathbf{r}_\mathbf{P} - \mathbf{r}_\mathbf{P}^*(t))^T (\mathbf{r}_\mathbf{P} - \mathbf{r}_\mathbf{P}^*(t)) + \beta (\dot{\mathbf{r}}_\mathbf{P} - \dot{\mathbf{r}}_\mathbf{P}^*(t))^T (\dot{\mathbf{r}}_\mathbf{P} - \dot{\mathbf{r}}_\mathbf{P}^*(t)) \Big)\, \mathrm{d}t, \qquad \min_{\mathbf{b}\in\mathscr{R}^k} J \quad \text{s.t.,} \quad \mathbf{F}(\mathbf{q},\dot{\mathbf{q}},\mathbf{b}) = \mathbf{0}. \tag{1}$$

Here, $\mathbf{q} \in \mathscr{R}^n$ is a vector of state variables and $\mathbf{b}$ denotes a vector of discretized input functions (e.g., driving torques or motor driving voltages). Symbol $\mathbf{r}_\mathbf{P}(\mathbf{q})$ denotes a dynamic response from the model for a given input $\mathbf{b}$. Once $\mathbf{b}^{\text{opt}}$ is computed, it can be used to recreate feedforward signal $\mathbf{u}_{ff}$.

Subsequently, the computed input signal is fed to the hardware. The measurement circuit with the highlighted mechanism on which the experiment has been carried out is presented in figure 2a. The multibody system constitutes a five-bar linkage actuated by two DC motors with a gear transmission. The device has a variety of sensors, such as a tachometer, potentiometer, and encoder. Its motors are controlled by supplying the appropriate voltage from the data acquisition (DAQ) card and the amplifier. The hardware is controlled via Matlab Simulink environment installed on a PC.



(a) (b)

Figure 2: (a) Five-bar linkage driven by two DC motors and (b) measured trajectories of the end-effector

3 Preliminary experimental results and conclusions

The input signal $\mathbf{u}_{ff}$ and reference trajectory $\mathbf{y}_d$ have been found by solving the problem (1) offline, i.e., in a simulation environment. Recorded solution has been provided as a nominal input signal to the control system presented on fig. 2a. The internal variables (motor shaft orientation) have been measured via embedded encoder and utilized to compute the Cartesian coordinates of the end-effector **P**. Specified procedure has been executed on the hardware in two distinct modes: adjoint-based (feedforward–feedback) and PD controller-only (sole feedback action). The response of system has been measured and compared between both cases by computing integral squared error (ISE) index:

$$ISE = \int_0^{t_f} (\mathbf{r}_\mathbf{P}^{\text{meas}} - \mathbf{r}_\mathbf{P}^{\text{ref}})^T (\mathbf{r}_\mathbf{P}^{\text{meas}} - \mathbf{r}_\mathbf{P}^{\text{ref}})\, \mathrm{d}t. \tag{2}$$

The results are depicted on figure 2b. One can see that the ISE measure is lower in the case of a model-based approach. Furthermore, the presence of nominal input (feedforward) leads to much lower magnitudes of the feedback signal (e.g., a PD controller), which only has to correct for the disturbance and discrepancies between the object and the model.

Acknowledgments

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