Data-Driven Prediction of Forced Vibrations: Hybrid Modelling in Frequency and Time Domain

Tobias Westmeier[1], Daniel Kreuter[1], Simon Bäuerle[2], Hartmut Hetzler[2]

[1] Corporate Research
Robert Bosch GmbH
Robert-Bosch-Campus 1, 71272 Renningen, Germany
[tobias.westmeier, danielchristopher.kreuter]@de.bosch.com

[2] Mechanical Engineering, Engineering Dynamics
University of Kassel
Mönchebergstr. 7, 34125 Kassel, Germany
[bauerle, hetzler]@uni-kassel.de

EXTENDED ABSTRACT

1 Introduction

The use of Neural Networks (NN) for identifying and predicting nonlinear vibration behaviour is an exciting field of research, given in a vast number of physical and engineering applications. In particular, several pure Deep Learning (DL) based architectures have been proposed for time series prediction due to the rapid progress in NN research [1, 2, 8]. Once trained, this NN approach enables low-resource, real-time predictions of system responses. Unfortunately, studies towards the practical application of technical systems are scarce. Here, data-driven approaches aim at minimizing the testing time and cost by identifying the system behaviour based on a small number of measurements to predict future ones. While NN architectures perform well for many applications, they often lack generalization capability for unseen input data and longer prediction lengths, due to error accumulation over time.

Furthermore, they may become problematic when depicting the dynamic oscillating behaviour in forced nonlinear systems [3, 4, 5]. Therefore, we explore a hybrid modelling approach to approximate the nonlinear transfer behaviour between the external excitation and the system response. The main building blocks are the linear transfer function in frequency domain to include the linearized oscillating behaviour of our system into the architecture and an Autoregressive Neural Network (hybrid-ARNN) to account for nonlinear influences. As Network architectures, we compare simple ARNN and utilize modern Long-Short-Term Memory Networks(LSTM) and Gated Recurrent Units(GRU). While ARNN are very similar to state space models, LSTM and GRU provide an internal memory to approximate the system state [6, 7]. Therefore, first we approximate the linear transfer function to the given data in a two-step process. Afterwards, the neural network is trained with the nonlinear system response data using the linear solution and the external excitation as input in an autoregressive setting:

$$\hat{y}(t) = \mathbf{y_l}(t) + f_{\boldsymbol{\phi}}(\mathbf{x}^s(t), \mathbf{y_l}^s(t), \hat{\mathbf{y}}^s(t-1)), \qquad (1)$$

where $\hat{y}(t)$ describes the predicted system state at timestep t while $\mathbf{y_l}(t)$ the corresponding linear solution. The external excitation is defined as $\mathbf{x}^s(t)$ where s indicates a sequence (last s time steps). The previous linear and nonlinear system states are defined as $\mathbf{y_l}^s(t)$ and $\hat{\mathbf{y}}^s(t)$. Lastly, $f(\cdot)$ defines the mapping by the NN with architecture-specific parameters $\boldsymbol{\phi}$. We compare ARNN, which is easier to handle and can contain basic properties such as symmetry, to LSTM and GRU, which have internal memory to save information over time [9]. Figure 1 shows exeplarically our hybrid ARNN architecture.

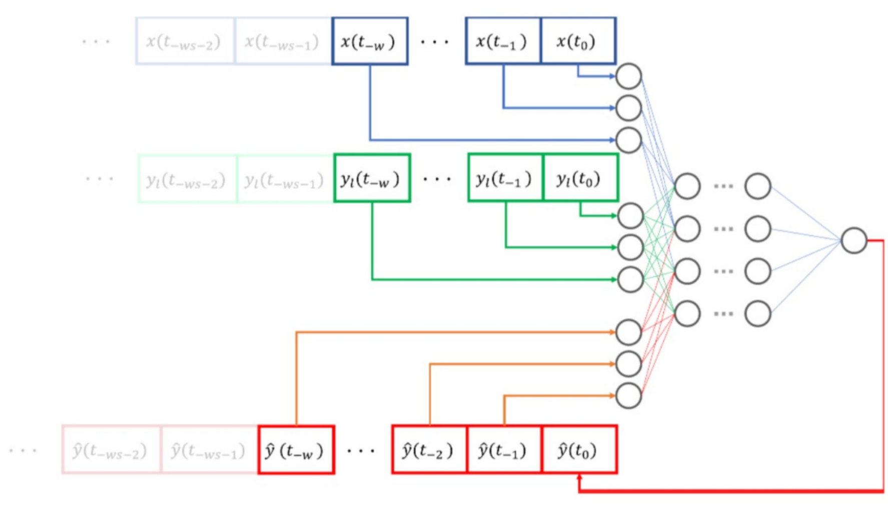

Figure 1: Hybrid-ARNN architecture to predict forced vibrations

All models are trained using adaptive gradient decent techniques in either closed-loop or open-loop environments. We perform a sequential probabilistic optimization to find ideal hyperparameter combinations for training and interference.

2 Results and Discussion

We investigate oscillations of the DUFFING equation $\ddot{x}+2D\dot{x}+x+\alpha x^3=f(t)$ and an automotive use case based on measurement data. DUFFING system data is synthesized with generic external excitations $f(t)$ (white noise, sweep signals, sine functions) using a DORMAND-PRINCE integration method. The automotive use case extends this one-dimensional theoretical case towards a multidimensional excitation and response for real-world training and validation data from testing tracks. We investigate multiple regularization techniques and show a clear dependency in accuracy due to specific hyperparameters. Our examples emphasize that the proposed hybrid-ARNN achieves significantly higher prediction accuracy than classical linear methods and converges faster than pure ARNN during training. Furthermore, ARNNs train more quickly and achieve superior accuracy in time and frequency domain. At the same time, LSTM and GRU suffer from a recency bias in training and cannot extrapolate to different excitation signals, especially on long excitation signals. Some preliminary results are shown in Figure 2, showing a clear advantage of our models compared to classic linear approaches.

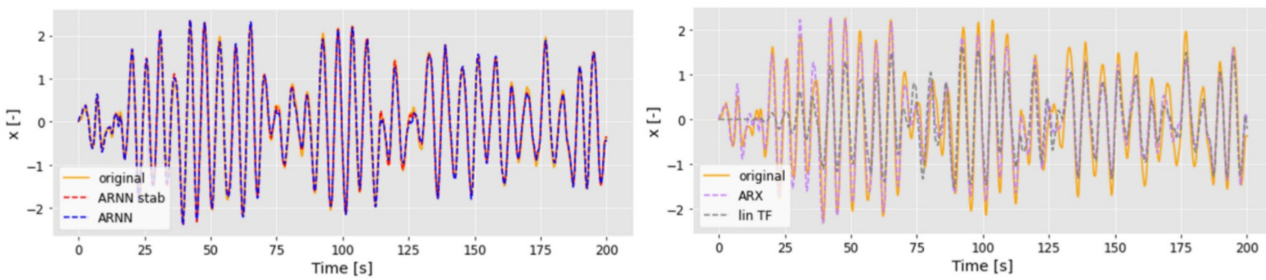

Figure 2: Time series results of the considered linear models (right) compared to the proposed nonlinear ARNN model (left) [6]

Furthermore, we show that a unique penalty formulation applied to the weights significantly reduces the training parameters' sensitivity. Our novel workflow defines a suitable approach for predicting forced vibrations in academic examples and for a real-world case based on multidimensional measurement data.

References

[1] H. Hewamalage et al. Recurrent Neural Networks for Time Series Forecasting: Current status and future directions. International Journal of Forecasting, 37: 388-427, 2021

[2] P. Lara-Benítez et al. An Experimental Review on Deep Learning Architectures for Time Series Forecasting. International journal of neural systems, 31, 2021.

[3] Y. Park et al. Recurrent Neural Networks for Dynamical Systems: Applications to Ordinary Differential Equations, Collective Motion, and Hydrological Modeling, arXiv, 2022

[4] L. Dostal et al. Predictability of Vibration Loads From Experimental Data by Means of Reduced Vehicle Models and Machine Learning, IEEE Access, 8: 177180-177194, 2020

[5] E. Diaconescu. The use of NARX neural networks to predict chaotic time series, WSEAS Transactions on Computer Research, 3, 2008

[6] T. Westmeier et al. Data Driven prediction of forced nonlinear vibrations using stabilised Autoregressive Neural Networks, Proceedings of Applied Mathematics and Mechanics, 2022

[7] Kemmler et al. Accelerated vibration testing: implementation of soft sensors for shaker profile derivation, Proccedings of ISMA, 2022.

[8] D. Salinas et al. DeepAR: Probabilistic forecasting with autoregressive recurrent networks, International Journal of Forecasting, 36, 2020

[9] K Greff et al. LSTM: A Search Space Odyssey, Transactions on Neural Networks and Learning Systems, 28, 2222–2232, 2017