Combined Optimal Control and Design of Flexible Multibody Systems

Daniel Lichtenecker[1], Karin Nachbagauer[2,3]

[1] Technical University of Munich, Germany
TUM School of Engineering and Design
Department of Mechanical Engineering
Chair of Applied Mechanics
Munich Institute of Robotics and Machine Intelligence (MIRMI)
daniel.lichtenecker@tum.de

[2]Faculty of Engineering and Environmental Sciences
University of Applied Sciences Upper Austria
Stelzhamerstraße 23, 4600 Wels, Austria
karin.nachbagauer@fh-wels.at

[3]Institute for Advanced Study
Technical University of Munich
Lichtenbergerstraße 2a, 85748 Garching, Germany

EXTENDED ABSTRACT

1 Introduction

The increasing industrial relevance of high-end solutions that fulfill a wide range of requirements demands the consideration of novel approaches at an early stage of virtual product development. For instance, finding an optimal control of highly flexible mechanical systems under consideration of final constraints is essential to perform a manipulation according to *a priori* defined tasks [1]. In addition, innovative lightweight construction requires novel approaches in the field of structural optimization. In a recent work [2], a comprehensive review of current research activities in the field of structural optimization is shown. Both, structural optimization and optimal control problems have to be adressed by engineers, but these two challenges are usually considered independently.

In this paper, the focus is on combined optimal control and structural optimization for highly flexible multibody systems. The idea of coupling both optimization tasks is promising to provide the best possible mechanical structure regarding an optimal control problem. An example of such an optimization problem is the mass minimization of a flexible robot system, where the robot is driven by torques and the tool-center point (TCP) must reach a defined state at the final time.

2 Flexible Multibody Formulation

Multibody systems with flexible components are often underactuated systems and the optimal control problem becomes more complicated in comparison to full actuated systems [3]. In this paper, flexible bodies are modeled by using the absolute nodal coordinate formulation (ANCF) as proposed by Omar and Shabana [4]. Considering a Selective Compliance Assembly Robot Arm (SCARA) as shown in Fig. 1(a), the state equations are given by a highly nonlinear first-order differential system

$$\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\boldsymbol{\omega}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{Q}_u + \mathbf{Q}_d - \mathbf{Q}_k \end{pmatrix}, \tag{1}$$

where the SCARA is formulated with a minimal set of generalized coordinates $\mathbf{q}$. Generalized velocities are denoted as $\boldsymbol{\omega} = \dot{\mathbf{q}}$ and $\mathbf{M}$ is the constant mass matrix, i.e. the mass matrix depends only on geometric and material parameters. Moreover, generalized forces according to external applied torques $\mathbf{Q}_u$, damping forces in the joints $\mathbf{Q}_d$ and elastic forces $\mathbf{Q}_k$ are considered.

3 Problem Definition

Structural optimization problems can be treated with so-called weakly and fully coupled methods [5]. Weakly coupled methods are based on equivalent static loads while fully coupled methods incorporates the dynamics of the system into the optimization process. The focus of this paper is to extend fully coupled methods in order to embed the optimal control of flexible multibody systems. Considering both optimization strategies leads to a combined set of optimization variables including the control formulation and the parameterized multibody system. The control is formulated as proposed in [1] with $\mathbf{u} = \mathbf{C}\bar{\mathbf{u}}$, where $\mathbf{C}$ is an interpolation matrix and $\bar{\mathbf{u}}$ is a set of discretized grid nodes. Assuming that the ANCF elements of the multibody system are parameterized with the height $\mathbf{h}$ and the width $\mathbf{w}$, the set of optimization variables is defined by $\mathbf{z}^\top = (\mathbf{h}^\top, \mathbf{w}^\top, \bar{\mathbf{u}}^\top)$. Minimizing the mass of the system is a common approach in structural optimization and is here extended to optimize the control as well. The nonlinear programming (NLP) problem can be formulated by

$$\min_{\mathbf{z}} m(\mathbf{z}) \tag{2}$$

s.t.

$$\mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} \tag{3}$$

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{4}$$

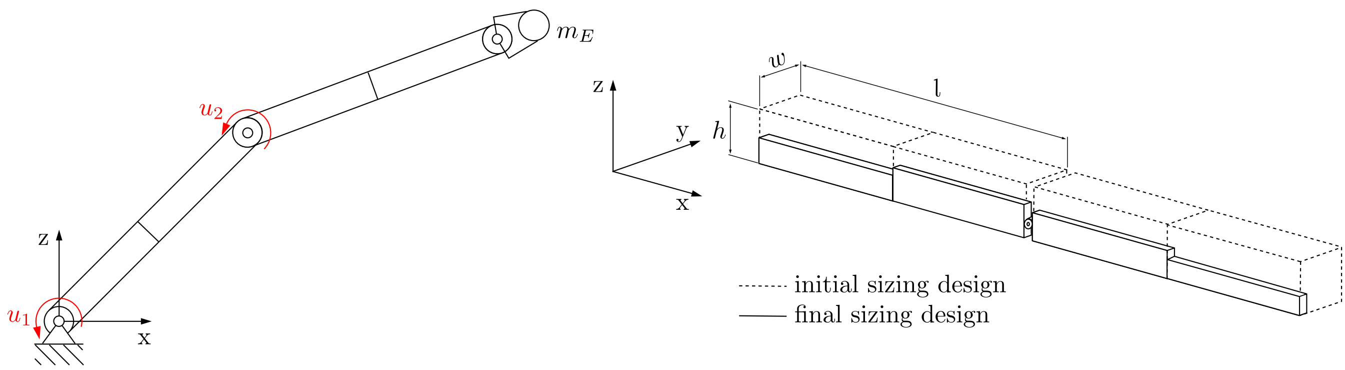
$$\mathbf{x}(t_f) = \mathbf{x}_f \tag{5}$$

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \tag{6}$$

with special attention to a state constraint at the fixed final time t_f. The NLP can be solved with classical direct optimization methods. These methods are based on first-order gradients of the optimization problem. Thus, the accurate and efficient computation of gradients takes a key role in the optimization process. In the last few decades, sensitivity analysis based on the adjoint method has become increasingly important, as e.g., [6]. In this paper, an adjoint approach is employed for the efficient computation of gradients. This approach is especially advantageous for problems where the design space is huge because the adjoint system does not depend on the number of design variables [7].

4 Example

In this paper, the combined optimization problem is addressed to minimize the mass of a SCARA as shown in Fig. 1(a) inspired from [8]. The dynamical behavior of the two-arm robot is described with the state equations in Eq. (1), where each arm is divided into two ANCF elements. This formulation leads to 8 sizing design variables and in addition each control is discretized with 10 grid nods. Material properties are set to $E = 3\text{e}9\ \text{N/m}^2$ and $\rho = 1300\ \text{kg/m}^3$ for the Young's modulus and the density, respectively. The length of both arms is $l_1 = l_2 = 1$ m and an additional mass $m_E = 1$ kg is attached at the TCP. The final time is set to $t_f = 3$ s, where the position and velocity of the TCP is prescribed at the final time. As an initial guess, the assumption



(a) (b)

Figure 1: (a) SCARA in a general undeformed configuration and (b) initial and final sizing design of the ANCF elements

for all geometric design variables are defined as $h_i = w_i = 50$ mm and the grid nodes parameterizing the control are set to $\bar{\mathbf{u}} = \mathbf{0}$ Nm. Figure 1(b) illustrates the optimal sizing of the ANCF elements to minimize the mass with respect to the constraints in Eqs. (3)-(6). With the final sizing design and the corresponding optimal control, the SCARA undergoes a large deformation during manipulation from the initial to the final state. This example demonstrates the combined optimization problem formulated for a highly flexible multibody system. The approach is promising to provide the best possible mechanical structure together with the required control history in one optimization.

Acknowledgments

Daniel Lichtenecker and Karin Nachbagauer acknowledge support from the Technical University of Munich – Institute for Advanced Study.

References

[1] D. Lichtenecker, D. Rixen, P. Eichmeir and K. Nachbagauer. On the Use of Adjoint Gradients for Time-Optimal Control Problems Regarding a Discrete Control. Multibody System Dynamics (in revision), 2022.

[2] V. Gufler, E. Wehrle and A. Zwölfer. A review of flexible multibody dynamics for gradient-based design optimization. Multibody System Dynamics, 53:379-409, 2021.

[3] R. Seifried. Dynamics of Underactuated Multibody Systems. Springer, Switzerland, 2014.

[4] M.A. Omar and A.A. Shabana. A two-dimensional shear deformable beam for large rotation and deformation problems. Journal of Sound and Vibration, 243:565-576, 2001.

[5] E. Tromme, O. Brüls and P. Duysinx. Weakly and fully coupled methods for structural optimization of flexible mechanisms. Multibody System Dynamics, 38:391-417, 2016.

[6] A. Held, On design sensitivities in the structural analysis and optimization of flexible multibody systems. Multibody System Dynamics, 54:53-74, 2021.

[7] A.E. Bryson and Y.C. Ho. Applied Optimal Control. Hemisphere, Washington, DC, 1975.

[8] S. Oral and S. Kemal Ider. Optimum design of high-speed flexible robotic arms with dynamic behavior constraints. Computers & Structures, 65:255–259, 1997.