Teaching Multibody Dynamics: from Rigid to Flexible Systems

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EXTENDED ABSTRACT

1 Introduction

Today, many universities offer courses on multibody dynamics as part of their curricula. In the field of mechanical engineering, multibody dynamics is relevant to study the motion of complicated machines and mechanisms and the associated. For many applications, it is sensible to assume that all bodies in a multibody system are rigid. For this reason, many engineering courses on multibody dynamics concern the modelling and simulation of rigid multibody systems. Courses dedicated to flexible multibody dynamics seem to be much scarcer.

In the last decade, the University of Twente (UT) has seen a growing interest in flexible multibody systems from researchers, students and industrial partners. For this reason, a dedicated course on flexible multibody dynamics (FMBD) was introduced for Master students at the UT in 2016. The course is based on the floating frame of reference formulation (FFF), in which a body's local elastic deformation is described based on a linear finite element (FE) model of the body. After several adjustments, the FMBD course is now running very successfully.

The goal of the course is to make students familiar with the theory related to the floating frame formulation and to offer them experience with the numerical implementation of this formulation in Matlab or Python. The course overview of several weekly learning modules was motivated in a contribution to the 2022 International Symposium on Modal Analysis [1]. Course material such as video lectures, tutorial problems and basic numerical codes are shared online [2]. In this way the authors wish to contribute to the successful teaching of FMBD by members of the engineering dynamics community.

The purpose of this work is to share important insights for teaching flexible multibody dynamics. In particular, it is explained what theoretical formulation is used and how to ensure that students are able to work on the numerical implementation of this formulation in parallel to the lectures. Didactical considerations related to student engagement and student learning are also presented. The focus is on the smooth transition from rigid to flexible multibody systems.

2 Prerequisites: rigid multibody dynamics, modal analysis and model order reduction

The floating frame of reference formulation is introduced for 2D rigid multibody systems. The generalized coordinates $\mathbf{q}$ describe the absolute position and orientation of each body's centroidal frame j relative to the inertial frame O. Kinematic constraint equations $\boldsymbol{C}$ describe rotational or translational joints at interface points and are formulated in the following form:

$$\boldsymbol{C}(\mathbf{q}) = \mathbf{0}. \tag{1}$$

In learning modules 1 and 2, kinematically driven systems are considered. It is explained how driving constraints can be formulated, how to solve the resulting nonlinear position problem using the Newton-Raphson method, and how to solve for the generalized velocities and accelerations from the velocity equation $\boldsymbol{\nu}$ and acceleration equation $\boldsymbol{\gamma}$:

$$\boldsymbol{C}_{\mathbf{q}}\dot{\mathbf{q}} = \boldsymbol{\nu}, \quad \boldsymbol{C}_{\mathbf{q}}\ddot{\mathbf{q}} = \boldsymbol{\gamma}, \tag{2}$$

in which $\boldsymbol{C}_{\mathbf{q}}$ denotes the Jacobian matrix of the constraint equations. In learning modules 3 and 4, the system's equation of motion in Lagrange multiplier form is presented. It is explained how to retrieve reaction forces from the Lagrange multipliers. For systems that are not kinematically driven, the constrained equations of motion in augmented form are derived:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{C}_{\mathbf{q}}{}^{T} \\ \boldsymbol{C}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{a} \\ \boldsymbol{\gamma} \end{bmatrix}, \tag{3}$$

in which $\mathbf{M}$ is the system's mass matrix, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers and $\mathbf{Q}_a$ is the vector of generalized forces. Basic numerical methods are presented for the time integration, as well as for satisfying the kinematic constraints after the time integration.

In learning modules 5 and 6, the attention shifts from the rigid multibody system to creating a model of a single flexible body. To this end, it is assumed that students have prior knowledge about the finite element method and modal analysis. At the UT, the Bachelor programme in Mechanical Engineering contains both topics. Furthermore, specific Master courses on the finite element method and structural dynamics cover these topics in more detail.

For this reason, the FMBD course assumes that the students are able to derive a linear FE model of a single component in the following form:

$$\mathbf{M}_{FE}\ddot{\mathbf{u}} + \mathbf{K}_{FE}\mathbf{u} = \mathbf{F}, \tag{4}$$

in which $\mathbf{M}_{FE}$ and $\mathbf{K}_{FE}$ are the FE mass and stiffness matrix respectively, $\mathbf{u}$ is the vector of nodal degrees of freedom and $\mathbf{F}$ is the vector of nodal forces. For different types of boundary conditions (clamped, hinged, free), the modal analysis of the single flexible body is performed. It is shown how one obtains the body's eigenfrequencies and eigenmodes from the relevant eigenvalue problem. In order to reduce computational time, model order reduction methods are used that are based on the modal truncation of free boundary eigenmodes and on the Craig-Bampton method. The discussion of other reduction methods is outside the scope of this course. The reduction is denoted as follows:

$$\mathbf{u} = \mathbf{\Phi}_f\mathbf{q}_f, \tag{5}$$

in which $\mathbf{\Phi}_f$ is the reduction basis of flexible modes and $\mathbf{q}_f$ the vector of generalized flexible coordinates. From the free boundary eigenvalue problem, the vector of rigid body modes $\mathbf{\Phi}_r$ is obtained as well. The combination of rigid and flexible modes is denoted by $\mathbf{\Phi}$.

At this point a rigid multibody model of the entire system is available as well as a reduced order FE model of each individual flexible body.

3 Building the flexible model

In learning module 7, all the aspects that must be changed to include flexibility on the system level are discussed. Here, the focus is on allowing the students to start implementing flexibility into their numerical code fast. First, the constraint equations $\boldsymbol{C}$ are updated to account for the fact that the local position of an interface point with respect to a body's floating frame is no longer constant, but depends on the flexible coordinates $\mathbf{q}_f$. The Jacobian matrix of the constraint equations and the acceleration equation are updated accordingly.

Due to time constraints, no formal derivation of the constrained equations of motion is presented in this learning module. Instead, a more intuitive explanation is given of how the constrained equations of motion of a flexible system can be formulated. To this end, a reduction basis $\mathbf{\Phi}$ is used based on free boundary eigenmodes. It is shown that $\mathbf{\Phi}^T\mathbf{M}_{FE}\mathbf{\Phi}$ yields a diagonal matrix $\bar{\mathbf{M}}$ of which the upper 3×3 partition equals the rigid body mass matrix and that $\mathbf{\Phi}^T\mathbf{K}_{FE}\mathbf{\Phi}$ yields a diagonal matrix $\bar{\mathbf{K}}$ of which the upper 3×3 partition equals zero.

Now, it is simply suggested to formulate the constrained equations of motion using $\bar{\mathbf{M}}$ instead of $\mathbf{M}$, using the updated constraint equations and by subtracting $\bar{\mathbf{K}}\mathbf{q}$ from the vector of applied forces $\mathbf{Q}_a$. It must mentioned carefully that the resulting set of equations is strictly wrong, because some effects are still missing. However, it is possible for the students to start with the numerical implementation at this point.

In learning module 8, a more formal derivation of the constrained equations of motion is presented. The didactical advantage of the numerical implementation described above, is that by this experience, students are able to understand the presented derivation more easily.

It is explained that the nodal coordinates $\mathbf{u}$ in equation (4) should be interpreted as local displacements, described relative to the floating frame j. This is denoted as $\mathbf{u}^{j,j}$. However, the nodal accelerations $\ddot{\mathbf{u}}$ should be interpreted as absolute acceleration, described relative to the floating frame j. The is denoted as $\ddot{\mathbf{u}}^{j,O}$. Kinematic relations for the absolute position, velocity and acceleration of an arbitrary point on a flexible body are derived. With this, it is possible to express the nodal accelerations in terms of the absolute accelerations of the floating frame, local accelerations and relative accelerations in quadratic velocity terms. Upon substitution of these kinematic relations in the FE equations (4), the correct expression for the body's mass matrix is obtained, as well as the vector of quadratic velocity forces that was missing in the constrained equations of motion derived before. Finally, the correct expression for the vector of generalized applied forces $\mathbf{Q}_a$ is obtained using the model order reduction as described by (5). The change from the intuitive form of the constrained equations of motions to the correct one requires very limited changes to the numerical implementation.

In the full paper and corresponding presentation, the authors will present all details of the derivations related to the flexible model. Also practical didactical considerations are shared that are relevant for the teaching of these derivations during lectures.

References

[1] J.P. Schilder, K.L. van Voorthuizen, M.H.M. Ellenbroek. Best practices in teaching flexible multibody dynamics. In Proceedings of the International Conference on Noise and Vibration Engineering (ISMA). KU Leuven, Leuven, 2022.

[2] J.P. Schilder, K.L. van Voorthuizen, M.H.M. Ellenbroek. Flexible Multibody Dynamics Course Material. Online open access via: https://canvas.utwente.nl/courses/11408. University of Twente, Enschede, 2022.