Edge Effects in Mixed Boundary Co-rotational Beam Elements

Bram Seinhorst[1], Marijn Nijenhuis[1], Wouter Hakvoort[1]

[1]Faculty of Mechanical Engineering
University of Twente
De Horst 2, Enschede, The Netherlands
{b.seinhorst/m.nijenhuis/w.b.j.hakvoort}@uwente.nl

EXTENDED ABSTRACT

1 Introduction

Beam elements are widely used for the simulation of flexure mechanisms undergoing large deflections. The low number of associated degrees of freedom results in highly efficient simulations for complex mechanisms. The assumptions behind the beam elements are usually based on the slenderness of the beam. That is, the element has a length (L) that is much greater than the dimensions of the cross section (h). Therefore, the edge effects become negligible. However, for shorter beams these effects can have a significant influence on the stiffness and stress. Similarly, they can become significant when the cross section of the beam discontinuously changes, for example due to local reinforcement or notches in leaf springs [1].

Several methods exist for modelling the end effects in spatial beam elements. Yu and Hodges applied the variational asymptotic method (VAM) to approximate the internal energy of a spatial beam section up to a specific slenderness order (h/L) [2]. The results are a generalized Timoshenko and Vlasov model for spatial beams with arbitrary anisotropic cross section. Additionally, the warping field of the cross section is obtained as a function of the traditional beam curvatures.

Instead of using the variational asymptotic method, one may also consider additional degrees of freedom associated with the so-called eigenwarpings of the cross section, similar to those introduced by Bauchau [3].

In this work we will compare the cross sectional warping of VAM to the eigenwarpings of the cross sections and discuss the appropriate boundary conditions. Additionally, using a spatial co-rotational beam element with additional warping degrees of freedom, we will compare the performance of both methods to a 3D finite element model in COMSOL. As an example we will focus on a notched leaf spring, which can be used in exact constrained flexure mechanisms. An illustration of the notched leaf spring and the various occurring boundary conditions is given in Figure 1.

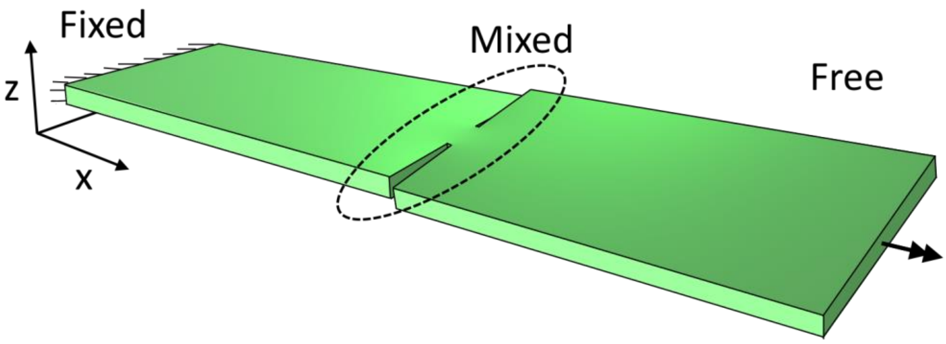

Figure 1: Boundary effects near a free, mixed and fixed boundary.

2 The cross sectional warping functions

The VAM can be used to construct a series of warping fields that yield asymptotically smaller contributions to the total internal energy as the aspect ratio of the beam increases. This results in the following expression for the cross sectional displacement w in the local frame:

$$w = \phi_0(y,z)\bar{u} + \phi_1(y,z)\bar{u}' + \phi_2(y,z)\bar{u}'' \dots \quad (1)$$

where $\bar{u}$ contains the usual coordinates of an elastic line associated with a beam. Alternatively, the displacement field can be expanded using the eigenwarping method as:

$$w = \tilde{w}(\bar{u}) + \psi_1(y,z)\eta_1 + \psi_2(y,z)\eta_2 \dots \quad (2)$$

Where $\tilde{w}(\bar{u})$ is the classical beam solution and $\psi_i(y,z)\eta_i$ are decaying boundary modes. I.e. $\eta_i \sim e^{\mu_i x}$. The functions $\phi_i(y,z)$

and $\psi_i(y,z)$ may be determined by discretizing the cross section and employing the finite element method.

3 Mixed boundary condition

We model the notched leaf spring in Figure 1 using two beam sections. The first section is fixed at the left side and a moment is applied on the right side of the second element. The interface (Γ) can be split into an area where the elements connect (Γ_u) and a section where the displacements are free (Γ_f) . This results in the interface constraint on Γ_u

$$w_L = w_R \tag{1}$$

Weakly enforcing this constraint using the test functions $t(y,z)$ results in:

$$\int_{\Gamma_u} t^T w_L \; dA = \int_{\Gamma_u} t^T w_L \; dA \quad \forall t \tag{1}$$

The final constraints can be obtained by substituting suitable test functions and either the VAM or the eigenwarping expansion.

4 Results

Figure 2 shows the torsional angle $\theta(x)$ and torsional strain $\kappa(x)$ for the notched leaf spring depicted in figure 1. As can be seen, there is sudden increase in rotation angle in the 3D finite element solution occurring at the notch. The traditional Vlasov beam model is not able to capture this discontinuity and therefore the leaf spring behaves significantly stiffer than the finite element solution. Once we give the cross section more freedom by including higher order warping solutions, it can be seen that the discontinuity behavior can be captured. Similar results can be obtained for bending and extensional stiffness.

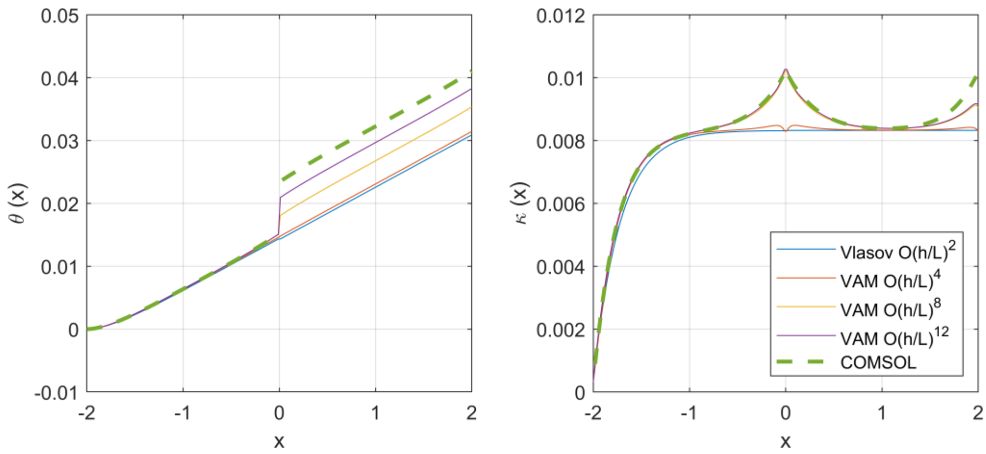

Figure 1: The torsional angle $\theta(x)$ and torsional strain $\kappa(x)$ in the notched leaf spring.

3 Conclusion

Edge effects can be significant for the stiffness of spatial beam elements, especially when subject to mixed boundary conditions. We show that by introducing cross sectional warping coordinates these effects can be captured with spatial beam elements.

References

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