A Symbolic Approach for Deriving Dynamics Equations for Multi-Link Flexible Manipulator System

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EXTENDED ABSTRACT

1 Introduction

The flexibility in manipulators are generally induced because of long links length compared to cross-sectional area or improper selection of material and high payload. These manipulators provides high payload to weight ratio, high operation speed, large workspace and reduces the overall weight of manipulator, due to these, the presence of flexible manipulators can not be ignored in many area like aerospace, medical and constructions industries etc. But problems arises when there is less positional accuracy due to structural vibrations induced because of elasticity [1]. These flexibilities made dynamics of flexible manipulators complicated compared to dynamics of rigid manipulators due to inclusion of extra coordinates [2].
Flexible manipulators acts as underactuated system as there is no actuation present for the extra coordinates. To define the deformation in these manipulators, several techniques are used like Finite Element Method (FEM), Assumed Mode Method (AMM) and Finite Segment Method (FSM) etc [2]. Mostly, studies are done in this area are numerical based. These studies are done for particular case which loose the generality of the model. Dynamic attributes of the manipulators counting stability, time response and vibration frequencies are deciphered based on a specific case. Besides, numerical frameworks should work utilizing numeric approximations, whose accuracy is restricted by the software. Some symbolic approach are also found where Lagrangian approach is used to derive the equations of motions [1,3,4]. In these studies, only single or two elements are considered for single link flexible manipulator. Accurate quantities can be acquired by holding the calculations in a symbolic structure. A distinctive feature of symbolic-based methods is the mathematically comprehensive output they generate, allowing the significance of individual terms, or group of terms, to be identified. This brings with it the opportunity to gain insights into the model that would otherwise not be possible. The existing symbolic approaches involves difficult calculus if number of degrees of freedom are more. Aforementioned problem have been taken as the primary objective in this work. A symbolic approach will open up the possibilities of analysing a framework in both new and fascinating ways. It may be seen that the pattern over the time has been away from completely numeric techniques for formulations towards those with a solid and complete symbolic flavor to them. This is expected to predominantly quick upgrades in computational power of computers.

2 Methodology

In this work, a new symbolic computational approach to derive the dynamics equations of motion for multi-link flexible manipulators system using finite element method (FEM) and decoupled natural orthogonal complement (DeNOC) matrices is proposed. The hybrid Euler-Lagrangian approach is used for this purpose. The proposed approach is able to derive dynamics equations with any number of elements per link. Matrix and vector related to each element are assembled easily using a connectivity matrix to get the global matrix and vector. In current study, links are discretized as two node beam elements. Deformations are considered in transverse directions only for current research but can also be extended to links having deformations like longitudinal and torsional etc. The accuracy of proposal approach is checked by considering the single-link and two-link flexible manipulator. In these simulations, two degrees of freedom are taken at each node, one is displacement and other is slope. Links are modelled using Euler-Bernoulli beam theory.

For dynamic modelling, DeNOC Matrices have been used to derive the equations of motion. The block diagram of approach is shown in the Fig. 1. In this figure $a_{i,i-1}$ is the positon vector form the origin of i^{th} link $(i-1)^{th}$ link, while Δ correspond to the slope at the tip of link. Rest symbols have their usual meaning if not mentioned. Assuming its deformation very small compare to the angular displacement of the hub and no shear deformation is there, then a flexible link can be taken as a "Euler-Bernoulli beam". To model the beam it is assumed that it has satisfied all the basic assumptions of the Euler beam theory. Finite element method is used to discretize the links deformation. Using this approach, one can vary the number of elements easily to get the desired dynamic equations. The main advantage of this approach compared to those available in literature is that current approach is completely based on the linear algebra theory. Apart from this, recursive algorithm which are computationally efficient and numerical stable can be written using this approach for forward and inverse dynamics.

3 Keywords

Flexible manipulators, Finite element method, DeNOC matrices, Symbolic computation.

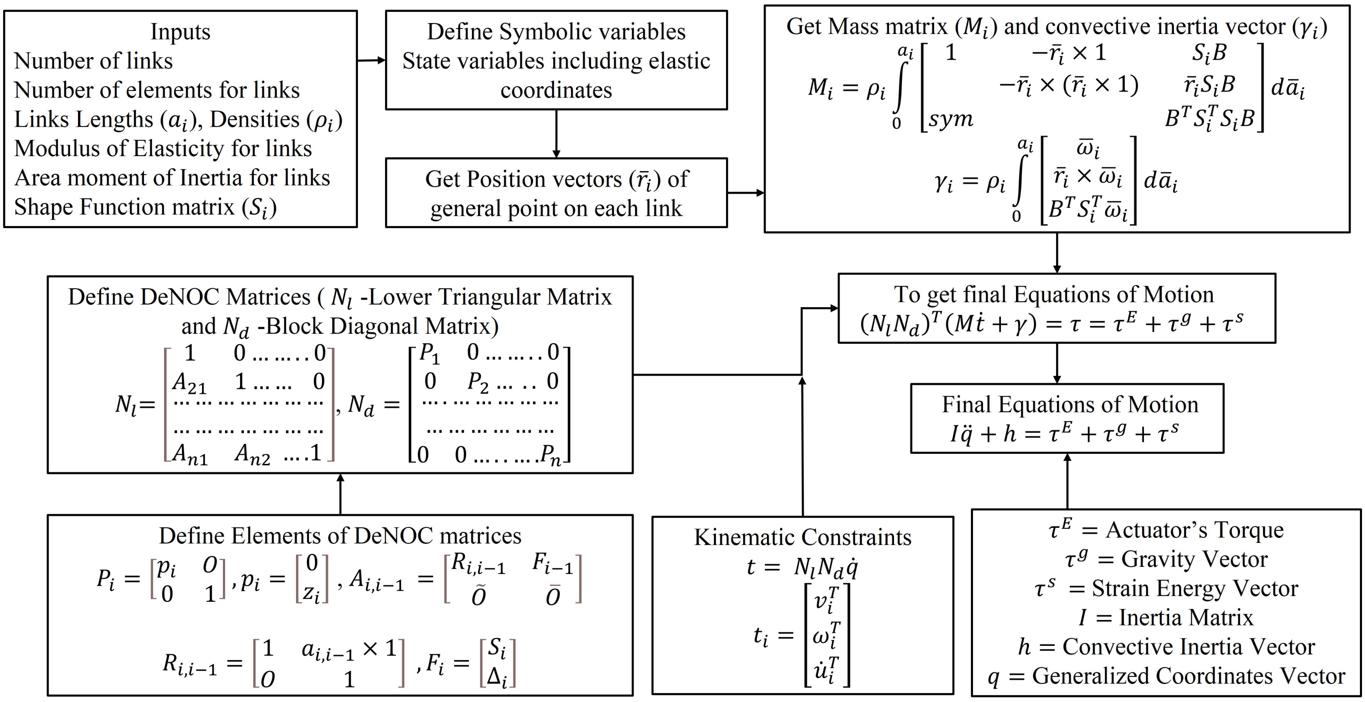

Figure 1: Block diagram for symbolic formulation

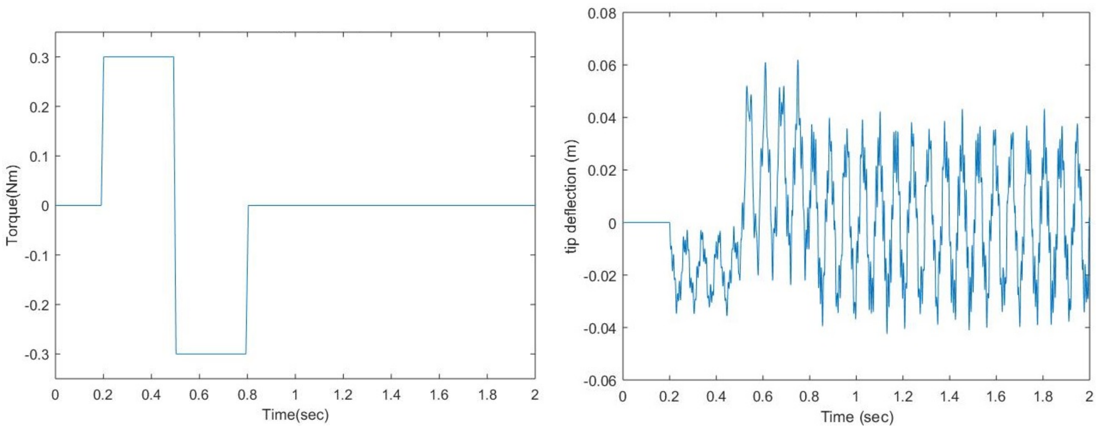

Figure 2: Input Torque and Tip Deflection for single flexible link

4 Results

The results for single and two link flexible manipulators are validated in frequency and time domain. The frequency domain validation includes the model frequencies and time domain validation includes the hub angle, hub angular velocity and tip deflection variation with respect to time. The results are found to be in a close agreement with literature. The results of single link are shown in Fig. 2, for link made of Aluminium with length 0.9m and area moment of inertia 5.1924×10^{11}. Similarly, simulation have been performed for two link flexible manipulator and results are validated with literature.

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