Dynamics analysis of a mobile crane with taking into account the hanged load eccentricity

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EXTENDED ABSTRACT

The method of modeling a load and a rope sling system has a significant impact not only on its dynamics but also on the dynamics of the crane. The literature is rather dominated by simplified models, which treat the load as a lumped mass or a rigid body hanged on one rope. It should be noted that in the second case, the assumption of a centric load suspension is frequently applied [1-4]. The mathematical model of a mobile crane with a rope sling system and a eccentrically hanged load (Fig. 1) is presented in the paper. The proposed model includes: crane suspension subsystem b, supporting structure c_m (consisting of a rotary column, two boom sections and telescopic boom section) and two load lifting subsystems $c_{a,\alpha}\big|_{\alpha\in\{1,2\}}$ (refer to hydraulic cylinders). The carried load is modelled as a box hanged eccentrically using rope sling system ($e_\alpha\big|_{\alpha\in\{x,y,xy\}} \in \{0, 0.1, 0.2\}\,\mathrm{m}$). The crane movement cycle is forced by rigid (Fig. 2a) or flexible (Fig. 2b) drives and the movement is divided into five phases, i.e. load lowering ($\mathbf{f}^{(d_3)}$), crane rotation ($\mathbf{t}^{(d_1)}$), load telescoping ($\mathbf{f}^{(d_4)}$), load lifting ($\mathbf{f}^{(d_2)}$), and load swinging.

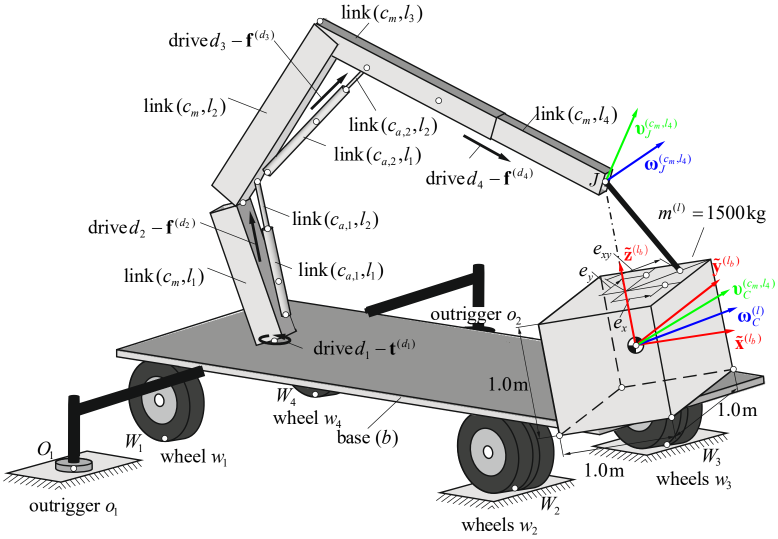


Figure 1: Model of the system

The motion of the crane is defined by the vector of generalized coordinates

$$\mathbf{q} = \left[\mathbf{q}^{(c)^T} \;\middle|\; \mathbf{q}^{(l)^T} \right] = \left[\mathbf{q}^{(b)^T} \quad \overline{\mathbf{q}}^{(c_m)^T} \quad \overline{\mathbf{q}}^{(c_{a,1})^T} \quad \overline{\mathbf{q}}^{(c_{a,2})^T} \;\middle|\; \mathbf{q}^{(l)^T} \right]^T, \tag{1}$$

where: $\mathbf{q}^{(c)}, \mathbf{q}^{(l)}$ define the motion of the crane and the load, respectively.

The dynamics equations of motion can be written in the following form

$$\begin{bmatrix} \mathbf{A} & \mathbf{C}^T \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \mathbf{f} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{c} \end{bmatrix}, \tag{2}$$

where: $\mathbf{A}$ is the mass matrix, $\mathbf{C}$ is the constraints matrix, $\mathbf{f}$ is the vector of the cut-joint reaction forces and driving forces and torque in the case of rigid drives, $\mathbf{F}$ is the vector of the right side of dynamics equations, $\mathbf{c}$ is the vector of the right side of constraints equations.

The kinetic energy of the load in relative motion (defined with respect to the tip of the telescopic boom, i.e. link (c_m,l_4) – Fig. 1) is determined by the following formula

$$\widehat{E}_k^{(l)} = \frac{1}{2} \widehat{\mathbf{q}}^{(l)^T} \tilde{\mathbf{H}}^{(l)} \widehat{\mathbf{q}}^{(l)}, \tag{3}$$

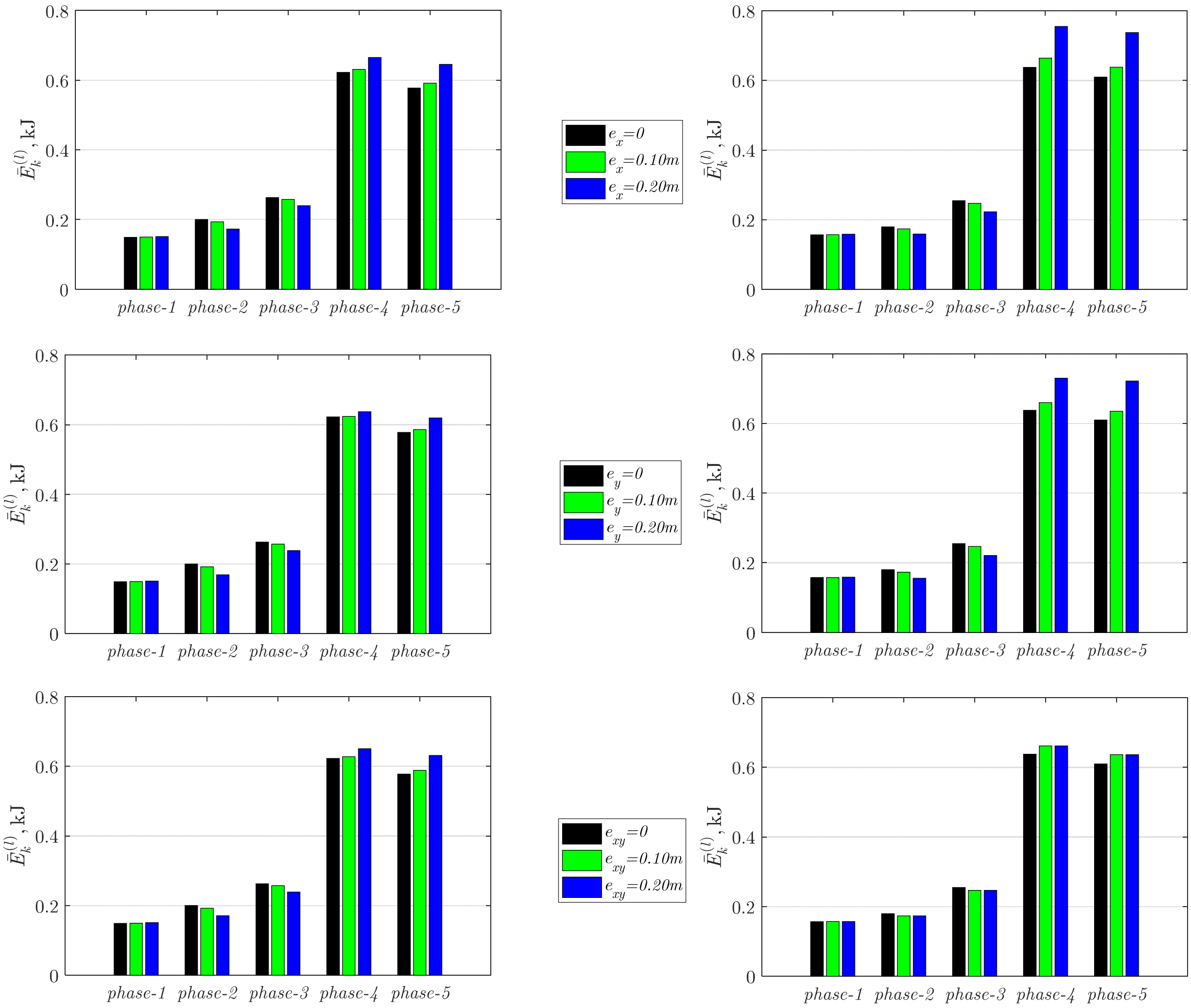
where: $\widehat{\mathbf{q}}^{(l)} = \begin{bmatrix} \boldsymbol{\upsilon}_C^{(l)} - \boldsymbol{\upsilon}_J^{(c_m,l_4)} \\ \boldsymbol{\omega}_C^{(l)} - \boldsymbol{\omega}_J^{(c_m,l_4)} \end{bmatrix}$, $\tilde{\mathbf{H}}^{(l)} = \begin{bmatrix} \mathbf{M}^{(l)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}^{(l)} \end{bmatrix}$, $\mathbf{M}^{(l)} = \mathrm{diag}\{m^{(l)}, m^{(l)}, m^{(l)}\}$, $\mathbf{I}^{(l)} = \mathrm{diag}\{I_x^{(l)}, I_y^{(l)}, I_z^{(l)}\}$.

The influence of the hanged load eccentricity on the relative kinetic energy of the load is estimated by integral mean value indicator

$$\bar{E}_k^{(l)}\Big|_{t\in\left[t_{iv}^{(\alpha)},t_{fv}^{(\alpha)}\right]} = \frac{1}{t_{fv}^{(\alpha)} - t_{iv}^{(\alpha)}} \int_{t_{iv}^{(\alpha)}}^{t_{fv}^{(\alpha)}} \widehat{E}_k^{(l)}(t)\mathrm{d}t\,, \tag{4}$$

where: $t_{iv}^{(\alpha)}, t_{fv}^{(\alpha)}$ define the time range of phase α.

A comparison of the integral mean value of the kinetic energy of the load in subsequent phases of the input movements for the system with rigid and flexible drives as well as for different values of the load eccentricities is shown in Fig. 2.



a) rigid drives b) flexible drives

Figure 2: Kinetic energy integral mean value of the load in phases of input movement

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