Online Data-Driven Modeling Of 2DOF Planar Robot Using Time-Delayed Dynamic Mode Decomposition

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EXTENDED ABSTRACT

1 Introduction

Traditionally, multibody system (MBS) dynamics models for robotic applications are derived from first principles. Physics-based representations may lead to ordinary differential or differential-algebraic formulations. MBS models extrapolate well by design and are usually preferred in the model-based control strategies. Data-driven techniques to identify dynamics directly from data are quickly growing, with a range of recent advancements. The emerging method called dynamic mode decomposition (DMD), seems to be a highly versatile and powerful approach to discover dynamics from time-series recordings, numerical simulations.

A key benefit of the data-driven DMD framework and variety of extensions is the simple formulation in terms of techniques well-known from linear algebra, even in the case when control signals are taken into account. An important extension of the original DMD approach employs time-delay embedding [1], which helps discover system dynamics from limited state measurements as it is the case in robotics. Time-delayed coordinates are built by stacking multiple time-shifted copies of the data into an augmented data matrix (called Hankel matrix). Subsequently, data-driven DMD identification is performed on top of the collected data.

In this paper we formulate an online version of the DMD approach, in which the model is updated as new measurements become available [2]. A novelty of the paper comes from using time-delay coordinates in online identification of the data-driven MBS model. We explore the effects of the number of time-delay coordinates on the quality of predicitions. The proposed identification method is demonstrated on a planar two-degree-of-freedom robotic system shown in Fig. 1 using data from a real plant experiment. The method will be succinctly introduced further in application to the mentioned dataset.

2 Online Dynamic Mode Decomposition With Time-Delayed Embeddings

The measurements that we use in this work are recordings of coordinates q_1, q_2 (Fig. 1) and signals u_1, u_2 applied to motors driving joints related to the q_1, q_2 respectively, when the mechanism's point, represented by a red dot in the figure, is programmed to move along a Lissajous curve (shown with a dashed line). The data is collected with a frequency of 500 Hz and structured as follows

$$\mathbf{X} = \begin{bmatrix} q_1(t_1) & q_1(t_2) & \dots & q_1(t_{n-1}) \\ q_2(t_1) & q_2(t_2) & \dots & q_2(t_{n-1}) \\ \dot{q}_1(t_1) & \dot{q}_1(t_2) & \dots & \dot{q}_1(t_{n-1})) \\ \dot{q}_2(t_1) & \dot{q}_2(t_2) & \dots & \dot{q}_2(t_{n-1})) \\ u_1(t_1) & u_1(t_2) & \dots & u_1(t_{n-1}) \\ u_2(t_1) & u_2(t_2) & \dots & u_2(t_{n-1}) \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{n-1} \\ | & | & & | \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} q_1(t_2) & q_1(t_3) & \dots & q_1(t_n) \\ q_2(t_2) & q_2(t_3) & \dots & q_2(t_n) \\ \dot{q}_1(t_2) & \dot{q}_1(t_3) & \dots & \dot{q}_1(t_n)) \\ \dot{q}_2(t_2) & \dot{q}_2(t_3) & \dots & \dot{q}_2(t_n)) \end{bmatrix}, \tag{1}$$

where $\dot{q}_1$, $\dot{q}_2$ are numerically found derivatives of q_1, q_2 and $t_i,\ i \in \{1, 2, \dots, n\}$ represents discrete time.

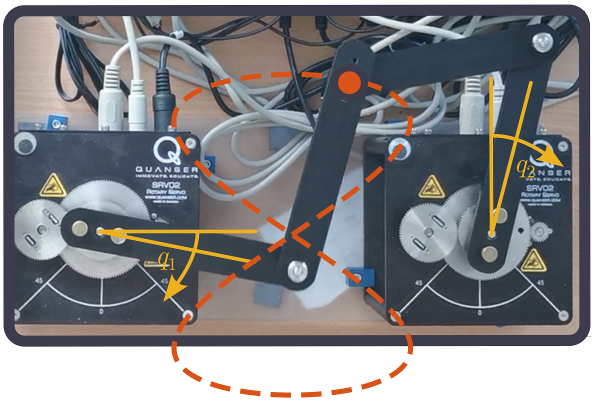

Figure 1: The identified five-bar linkage mechanism. Coordinates q_1, q_2 represent the controlled joints' rotation.

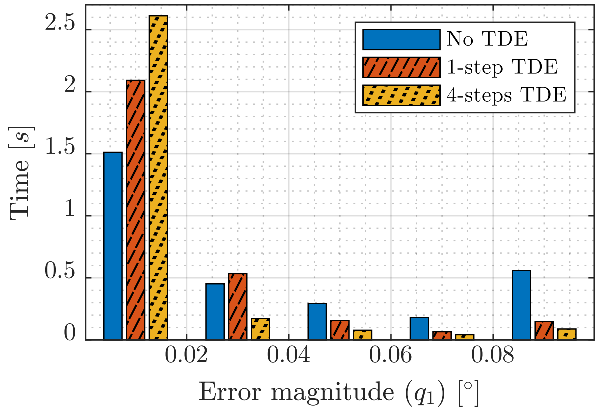

Figure 2: Histogram presenting q_1 prediction error for different time-delay embedding (TDE).

The fundamental idea beneath the DMD method is to find the best-fit matrix, which maps from $\mathbf{X}$ to $\mathbf{Y}$. If we conclude the best fit to be a least-squares problem solution, we can find the mapping followingly

$$\mathbf{Y} = \mathbf{A}\mathbf{X} \Rightarrow \mathbf{A} \approx \mathbf{Y}\mathbf{X}^{\dagger}, \tag{2}$$

where $\mathbf{X}^{\dagger}$ is the Moore-Penrose inverse of $\mathbf{X}$. In such a setting, we will find matrix $\mathbf{A}$, which can be used to predict the next state of the object based on the most recent measurement. This perfectly linear approximation might not be enough to predict the states firmly. There are multiple ideas to overcome the issue, e.g., the application of Sparse Identification of Nonlinear Dynamics Systems (SINDy) for optimal control of open-loop MBS [3], which could be translated to DMD as extending $\mathbf{X}$ with nonlinear functions of the measurements. Another idea, called time-delay embedding (TDE), is to augment $\mathbf{X}$, $\mathbf{Y}$ matrices with time-shifted copies of the measurements to turn them into the Hankel matrices

$$\hat{\mathbf{X}}_{\text{TDE}=l} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{n-l-1} \\ \mathbf{x}_2 & \mathbf{x}_3 & \dots & \mathbf{x}_{n-l} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_{l+1} & \mathbf{x}_3 & \dots & \mathbf{x}_{n-1} \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \hat{\mathbf{x}}_1 & \hat{\mathbf{x}}_2 & \dots & \hat{\mathbf{x}}_{n-1} \\ | & | & & | \end{bmatrix}, \quad \hat{\mathbf{Y}}_{\text{TDE}=l} = \begin{bmatrix} | & | & & | \\ \hat{\mathbf{y}}_1 & \hat{\mathbf{y}}_2 & \dots & \hat{\mathbf{y}}_{n-1} \\ | & | & & | \end{bmatrix}, \tag{3}$$

where l in $\hat{\mathbf{X}}_{\text{TDE}=l}$, $\hat{\mathbf{X}}_{\text{TDE}=l}$ means the number of shifted copies attached to the original matrices. If we set $l = 2$, it means that the model needs 3 last measurements to approximate the upcoming state.

Nevertheless, the matrices shortly rise to the dimensions counted in thousands, and computing them for every new measurement becomes computationally inefficient. To cope with the issue, it is possible to reformulate (2) to a recursive problem, as shown in [2]. If we introduce $\mathbf{A}_k = \hat{\mathbf{Y}}_k\hat{\mathbf{X}}_k^T\left(\hat{\mathbf{X}}_k\hat{\mathbf{X}}_k^T\right)^{-1} = \mathbf{Q}_k\mathbf{P}_k$ for k-th update, the formulas follows

$$\begin{aligned} \gamma_{k+1} = \frac{1}{1+\hat{\mathbf{x}}_{k+1}^T\mathbf{P}_k\hat{\mathbf{x}}_{k+1}}, \quad \mathbf{P}_{k+1} = \mathbf{P}_k - \gamma_{k+1}\mathbf{P}_k\hat{\mathbf{x}}_{k+1}\hat{\mathbf{x}}_{k+1}^T\mathbf{P}_k, \\ \mathbf{A}_{k+1} = \mathbf{A}_k + \gamma_{k+1}\left(\hat{\mathbf{y}}_{k+1} - \mathbf{A}_k\hat{\mathbf{x}}_{k+1}\right)\hat{\mathbf{x}}_{k+1}^T\mathbf{P}_k. \end{aligned} \tag{4}$$

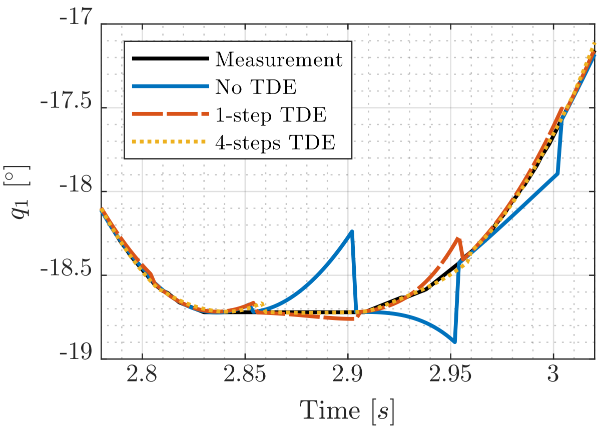

Figure 3: Measurement and predictions of q_1.

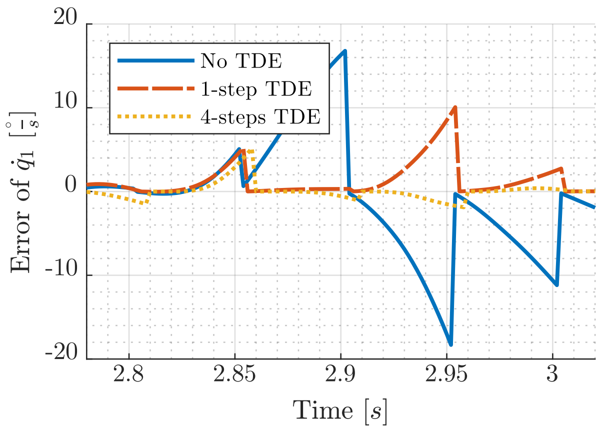

Figure 4: Prediction error of $\dot{q}_1$.

3 Sensitivity Of Results Relative To The Time-Delay Embedding

A natural extension to the recursive scheme is to apply a weighting to decay older measurements and build the model only from a number of the most recent snapshots (a window). For described dataset, we used weighting equal to 0.999 and a window of the duration of 1 second (the oldest data is weighted by $0.999^{500} = 0.6$). We build models without TDE and with 1-step or 4-step TDEs ($l \in \{1,4\}$) and ask them to predict 0.05 s (25 samples). In Fig. 2, it can be seen that by increasing TDE, the share of the error of the smallest magnitudes (between 0 and 0.02) increases, which translates to a more accurate prediction. Also, in Fig. 3 one can compare predictions of q_1 plotted over the real measurements – the 4-step TDE is qualitatively the same as measurement.

Longer time-delay embedding produces a more accurate system, but after reaching a particular limit, the predictions become unstable. Also, the prediction error of derivatives (Fig. 4) is substantially bigger. The authors would like to research further on these two issues.

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References

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