Phase portraits and bifurcations induced by dynamic friction models

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EXTENDED ABSTRACT

1 Introduction

In the literature, we can find a wide variety of friction models, and the choice between them is affected by several aspects of the analysis. Comparison of the different models usually focuses on numerical simulations [1, 2]. However, less attention is paid to qualitative analysis of the resulting dynamical systems from the perspective of the phase space. Our goal is to explore the nonsmooth dynamical phenomena and bifurcations, which can be observed in the phase space of mechanical systems with dynamic friction models.

2 Problem statement

Consider a mechanical system in the form

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} = F_F \cdot \mathbf{e}_1 + \mathbf{Q}(\mathbf{q},\dot{\mathbf{q}}), \tag{1}$$

where $\mathbf{q} = (q_1, \dots q_n) \in \mathbb{R}^n$ denotes the generalised coordinates, $\mathbf{M}$ is the mass matrix, and $\mathbf{Q}$ contains all generalised forces except for friction. We assume that the friction force F_F originates from a single planar frictional contact described by the relative tangential displacement and velocity $x \equiv q_1$, $v = \dot{x} \equiv \dot{q}_1$, respectively. Thus, $\mathbf{e}_1 = (1, 0, \dots 0)$ is the direction vector of the tangential motion of the contact. We consider the dynamics of (1) for different models of the friction force F_F. Our goal is to analyse the problem as a system of ordinary differential equations in the form $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ in the phase space $\mathbf{x} \in \mathbb{R}^m \subset \mathbb{R}^{2n}$ and utilise the toolkit of nonlinear and nonsmooth systems to explore the behaviour of the system.

3 Static friction models

In the usual context of the literature, the class of *static* friction models is defined slightly different ways. Now, we consider the models where an algebraic relation

$$f_F(F_F, F_N, x, v) = 0 \tag{2}$$

exists between the friction force F_F, the normal force F_N and the kinematic variables x, v of the tangential contact. (The normal force $F_N(\mathbf{q},\dot{\mathbf{q}})$ depends on the state variables of the mechanical system.) As an important sub-class, we can express the friction force at *slipping* in the explicit form $F_F = -\tilde{\mu}(v)F_N$. This case includes, most importantly, the classical Coulomb model and the Stribeck model (see e.g. [1, 2]), where $\tilde{\mu}(v)$ has a finite jump at $v = 0$.

4 Dynamic friction models with a single internal variable

In the literature, the concept of *dynamic* friction models applies to models with internal state variables of the contact. As a first extension of the static models, we can consider a *single* state variable z, and then, the relation (2) is extended to

$$f_F(F_F, F_N, x, v, z) = 0. \tag{3}$$

Moreover, the model should prescribe the time evolution of z, which is now considered in the form

$$\dot{z} = f_z(x, v, z). \tag{4}$$

The formulation (3)–(4) of dynamic friction models covers the widely used Dahl and LuGre models. Moreover, we can include the Generalised Maxwell-Slip (GMS) model [3] in the special case of a single contact element.

5 Minimal test example

For the analysis of the behaviour of the models, we can consider the single degree-of-freedom mechanical system, where the combination of (1) and (3)–(4) reduces into the system

$$\dot{x} = v, \qquad \dot{v} = f_v(x, v, z), \qquad \dot{z} = f_z(x, v, z) \tag{5}$$

in the phase space $\mathbf{x} = (x, v, z) \in \mathbb{R}^3$. Moreover, in the case of a system invariant to the translation x, the system (5) can be analysed in the phase plane $\mathbb{R}^2 \ni (v, z)$. When combining this minimal system with the different dynamic friction models (3)–(4), a detailed qualitative analysis is available to explore the structure and bifurcations in the dynamics. The results can be extended to systems of the general form (1).

6 Elements of discontinuous dynamics

The phase space of mechanical systems with planar frictional contacts can usually be described by piecewise smooth vector fields [4]. In the case of the Coulomb and Stribeck models, there is usually a *switching surface* (discontinuity set) at $v = 0$ with a finite jump in the vector field; systems in this class are often called *Filippov systems*. According to the transversal component of the vector field, the switching surface can be divided into two typical regions: In the *sliding region*, the trajectories of the slipping motion continue inside the switching surface corresponding to the dynamics of the static contact state (sticking or rolling). In the *crossing region*, the trajectories of slipping motion pass through the discontinuity, and the static contact state is not realisable. This structure is well-established in the literature.

However, the presence of dynamic friction models leads to additional phenomena and features in the phase space. With the Dahl and LuGre models, the vector field becomes *piecewise smooth continuous*, where the *derivative* of the vector field has a jump at the discontinuity. The structure of the trajectories is determined by the interaction of the discontinuity and the nonlinear structures of the friction models. One of the simplest scenarios can be seen in Figure 1. In the case of the GMS model, we can reformulate the original model [3] to get a piecewise smooth system with various types of discontinuities. As an interesting consequence, we can experience *sliding dynamics* corresponding to mechanical *slip*, which is the opposite scenario than what is experienced in the case of the Coulomb model.

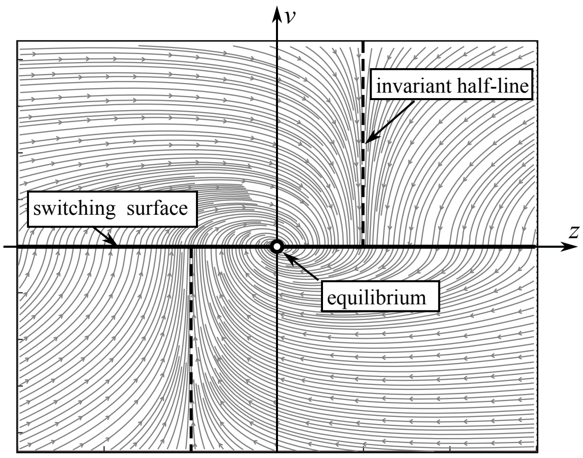

Figure 1: Phase portrait of a simple mechanical system with the Dahl friction model. By varying the mechanical system and the friction models, more complex situations occur where the different bifurcations can be studied.

7 Bifurcations and slip-stick transitions

Let us consider the stationary states of mechanical systems where the contact can be either in a slip or stick state. By varying the parameters, the system exhibits different types of bifurcations. One of the most significant bifurcations is when the stationary state switches from slip into stick state or vice versa. In the case of static friction models, the slip-stick transitions typically occur trough *tangency bifurcation*, which can also be observed also in spatial problems [5]. In the case of dynamic friction models, we encounter various degenerate cases of *transcritical bifurcations*, where slip-stick transitions occur. In the phase space, these bifurcations involve boundary equilibria, virtual equilibria and even pseudo-equilibria (in the case of the GMS model). The results provide a deeper insight into the phenomenon of slip-stick transitions.

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