Numerical Integration of Multibody Pantograph Model and Finite Element Catenary Model Using Newmark Method

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EXTENDED ABSTRACT

The pantograph-catenary system is an essential component in modern rail transportation as it provides electric current to the train. The dynamics of both the pantograph and the catenary must be compatible in order for steady current collection to occur and train operation to be smooth. A realistic mathematical model can provide insight into the system's behavior under different operating conditions and be used to optimize the system's performance. Additionally, the model can be used as a virtual testing tool to design and develop a pantograph-catenary system.

The catenary is mainly composed of contact and messenger wires that are linked by droppers. The structure is held in place by brackets and steady arms. The pantograph is a one-degree-of-freedom (DoF) linkage mechanism consisting of two closed loops representing the Watt six-bar mechanism. The mechanism is actuated using a pneumatic bellow which is simplified as a spring-damper system located at the base of the pantograph. Furthermore, the pantograph interacts with the contact wire using a registration strip supported on top of the pantograph using a spring-damper arrangement. The schematic of the pantograph-catenary system is shown in Figure 1.

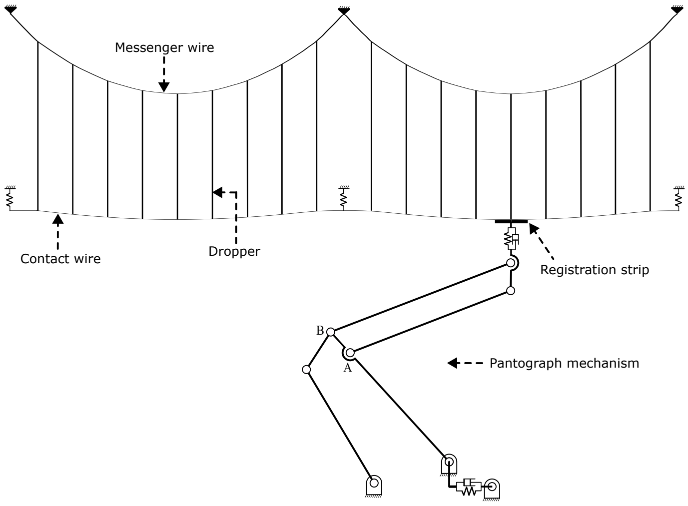

Figure 1: Schematic of the pantograph-catenary system

The contact and messenger wires are modeled as Euler-Bernoulli beam elements in the FE method, the droppers as bar elements, and the supports as spring-mass elements. The accurate simulation of the catenary involves capturing the dropper slackening phenomenon, which happens when the dropper exhibits almost zero stiffness when they are under compression during the pantograph passage and also the contact loss phenomenon. The dynamic model of the closed-loop mechanism of the pantograph is developed using the concept of the Decoupled Natural Orthogonal Complement (DeNOC) matrices [1].

In literature, different numerical integration methods have been utilized to solve the Differential Algebraic Equations (DAE) of the pantograph and the Ordinary Differential Equations (ODE) of the catenary in a co-simulation procedure designed to simulate their interaction [2, 3]. The methods used for solving the DAE include backward differentiation formula (BDF), implicit Euler or implicit Runge-Kutta methods. For the ODE, methods such as the Newmark and the HHT methods have been employed. The choice of method is dependent on the specific requirements of the simulation, such as accuracy, stability, and computational cost. In the present work, time integration of both models is carried out using the Newmark method [4]. An iterative scheme is used at every time step to incorporate the non-linearities due to dropper slackening, loss of contact and solve the non-linear algebraic equation resulting from discretizing the DAE of the pantograph using the Newmark formulas.

The discretized equation of motion of catenary at time t_{n+1} is shown in Equation 1. The same formulas are used to discretize the equation of motion of pantograph and position kinematic constraint equations as shown in Equation 2.

$$\mathbf{M}\ddot{\mathbf{d}}_{n+1} + \mathbf{C}\dot{\mathbf{d}}_{n+1} + \mathbf{K}\mathbf{d}_{n+1} = \mathbf{F}^s + \mathbf{F}^c_{n+1} + \mathbf{F}^d_{n+1} \tag{1}$$

$$(\mathbf{I}\ddot{\theta})_{n+1} + (\mathbf{\Phi}_\theta^T \lambda)_{n+1} + (\mathbf{C}\dot{\theta})_{n+1} - \boldsymbol{\tau}_{n+1} = \mathbf{0} \tag{2a}$$

$$\Phi(\theta_{n+1}) = \mathbf{0} \tag{2b}$$

In these equations, **M**, **C** and **K** are the global mass, damping and stiffness matrices of the catenary respectively, $\mathbf{d}_{n+1}$ and **F** represent the displacement of element nodes and external force vector respectively at time t_{n+1}, $\mathbf{F}^s$ contains the static forces acting on the both contact and messenger wire, $\mathbf{F}^c_{n+1}$ represents the contact force generated due to the interaction with the pantograph, $\mathbf{F}^d_{n+1}$ is added to include the balancing term to introduce the dropper slackening, **I** is the Generalised Inertia Matrix, **C** is the Matrix of convective inertia terms, $\boldsymbol{\tau}$ is the Generalized Force Vector, θ is the vector of generalized coordinates, $\mathbf{\Phi}_\theta$ is the constraint jacobian matrix and λ is the vector of lagrange multiplier.

The simulation of pantograph-catenary interaction was performed for vehicle running at 320 km/h for a time step size of 10^{-3} sec. for 15 spans of catenary. The key performance indices like contact force variation and pantograph displacement time history were evaluated as given in EN50318:2018 [5] standard. Figure 2 shows the drift of constraints when using the newmark scheme and BDF method with co-simulation procedure.

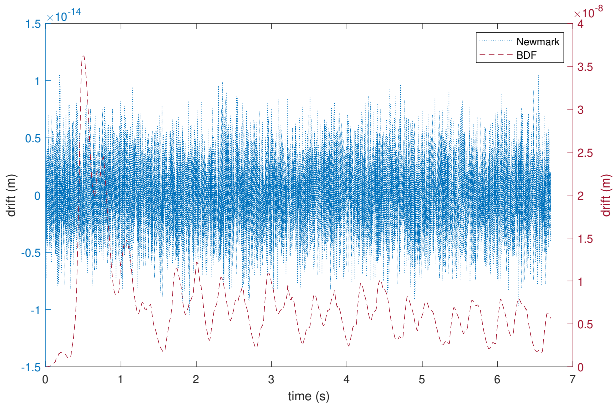

Figure 2: Drift of position-level kinematic constraints of the pantograph at point A

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