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Spatial beam element with third-order geometric stiffness formulation for improved mesh convergence

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EXTENDED ABSTRACT

1 Abstract

This paper presents the stiffness formulation of a beam element with the relevant third-order nonlinear geometric effects for relatively wide and thin rectangular beams, in particular when loaded in the plane and simultaneously deformed out of the plane. The formulation is based on Timoshenko beam theory with nonuniform torsion and Wagner effects. The derivation is carried out by means of the Hellinger–Reissner variational principle with custom interpolation functions. The element is incorporated into the generalized strain beam formulation for multibody systems. Numerical simulations of precision flexure mechanisms show that the use of a single third-order element per flexible member can already yield adequate performance, at a significant reduction of the necessary degrees of freedom and the computation time, compared with using multiple second-order elements in the generalized strain beam formulation.

2 Introduction

In the stiffness analysis of deforming structures, the third-order terms in the load–displacement relationship can have a significant effect, especially in highly slender structures such as thin and wide beams. Examples are the so-called sheet flexures or leafsprings in flexure mechanisms used for precision manipulation. To facilitate design optimization of flexure mechanisms, we aim to reduce computational effort by refining the element stiffness description, so that fewer beam elements are required for mesh convergence.

3 Methods

For this work, we derive a third-order geometric stiffness description for a numerical beam element in the generalized strain beam formulation [1]. A key aspect of the formulation is the use of generalized strain and stress variables, alongside the common nodal coordinates and loads. The generalized strains allow a natural specification of element rigidity and flexiblity. The generalized stresses can be used as Lagrange multipliers that enable better iteration convergence for slender structures [2]. The spatial beam element with second-order stiffness effects, as implemented in the software package SPACAR [3], is used as the benchmark in this work, together with a classical finite element solution in ANSYS.

The derivation of the third-order terms in the load–displacement relations is carried out by means of the Hellinger–Reissner variational principle with a specific set of physically-motivated displacement and stress interpolation functions. The stress interpolation functions are explicitly dependent on the displacement shape functions, which enables the use of low-order polynomials. The effects of axial elongation, bending, transverse shear, nonuniform torsion and Wagner terms are captured. An element with this third-order stiffness description has been implemented in SPACAR as well.

4 Results

Figure 1 (left) shows a cantilever beam with dimensions representative of precision applications of leafsprings. The displacement in y-direction at the free end has a prescibed value up to 15% of the length. In the deflected state, the linearized equilibrium

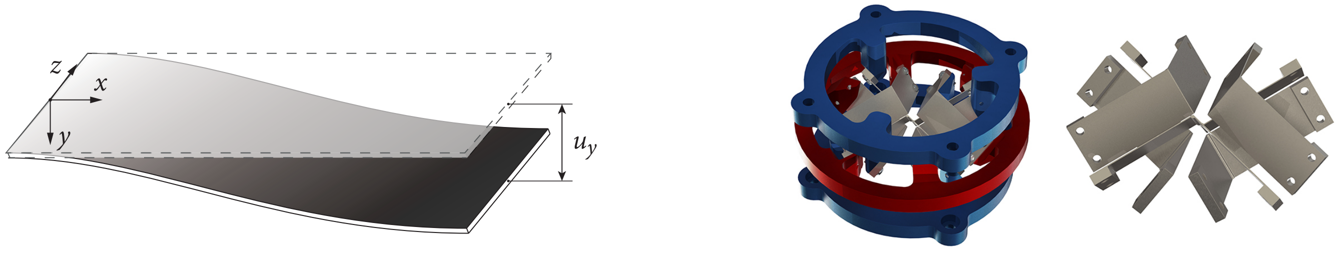

Figure 1: On the left, a cantilever beam with dimensions 100 mm by 30 mm by 2.25 mm. Nonzero displacement u_y is prescribed and the rotation about the z-axis is fixed to a value of zero. On the right, a flexure-based spherical joint with 12 leafsprings [4]. The end-effector is first tilted 30° (not shown) about a horizontal axis and then rotated a full revolution about the vertical axis.

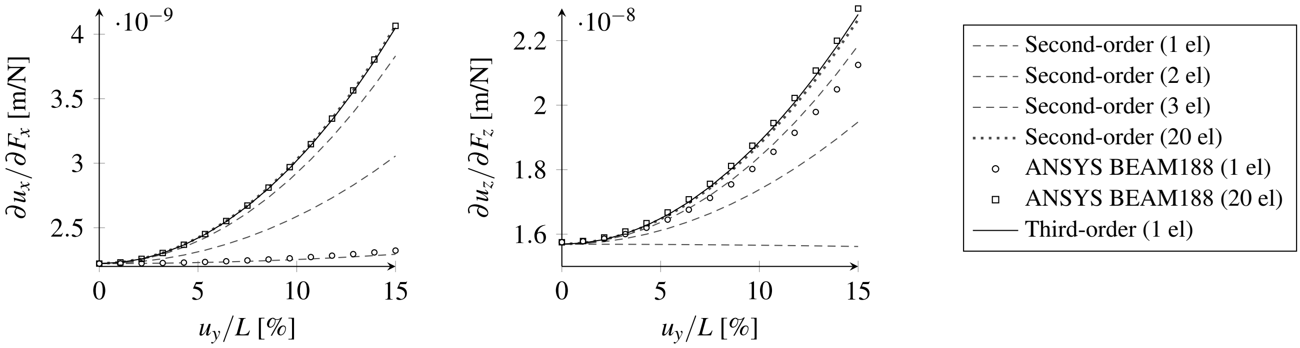

Figure 2: Two diagonal components of the compliance matrix at the free end of a cantilever beam with increasing deflection. Graphs show the new third-order element in SPACAR, the second-order element in SPACAR and the ANSYS BEAM188 element.

equations are used to determine the compliance (or flexiblity) matrix, relating increments in displacements and rotations to increments in forces and moments.

It can be seen in Figure 2 that more than three SPACAR elements with the second-order stiffness description are needed for reasonable convergence. The new element with third-order description performs better and lies within 0.8% of the converged value, represented by the 20-element SPACAR and ANSYS solutions.

As a second case, the flexure-based spherical joint of Figure 1 (right) is considered. This is a system with a larger number of flexible components: 12 leafsprings. It can be seen in Figure 3 that a single third-order element is only 2.3% off the converged solution with four second-order elements. Also, it presents a 68% reduction in computation time compared with the two-element second-order model of similar accuracy.

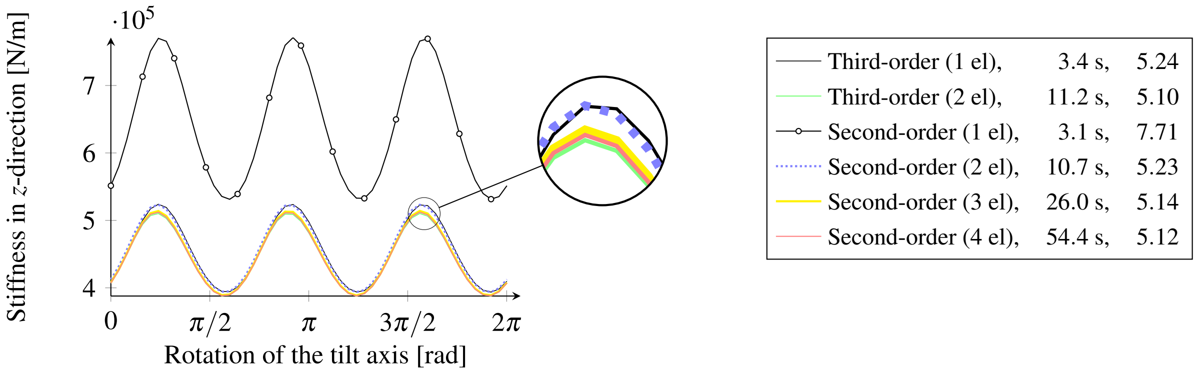

Figure 3: Vertical stiffness component (inverse of compliance) of the spherical ball joint, provided by models with the second-order and third-order beam element in SPACAR. The legend shows computation times in seconds and stiffness in 10^5N/m.

5 Conclusion

A new two-node spatial beam element with third-order geometric stiffness can be used for modeling wide and thin beams in the generalized strain beam formulation at lower computational cost compared with the second-order stiffness description. Using fewer elements, in many cases only one, outweighs the increased complexity per element, especially in systems with many flexible components.

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