

# Qualitative Dynamic Analysis of Mutual Interaction of Imperfections and Friction in Joints

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## EXTENDED ABSTRACT

### 1 Introduction

The existence of clearances or gaps in joints is a consequence of manufacturing tolerances and the physical functionality of the joints because the joint should allow relative motion between two bodies and the clearance is necessary for mobility. On the other hand, there are passive effects characterized mainly as friction during a relative motion. Two considered types of inner friction are dry and lubricated friction. The dry friction in joints is often accompanied by noise and wear [1] which can result in fatigue problems [2]. Also, the stick-slip effect can arise, and it results in sudden changes in the dynamical properties of mechanisms. From the viewpoint of basic research, the analysis and prediction of the stick-slip effect together with its utilization in the mechanism control model are still not well-researched areas.

The presented work deals with the research of possible mechatronic systems and devices whose motion and performance depend on the introduced friction (generally passive effects) on one side and joint clearances allowing more free motion on the other side (see Figure 1 for illustration of the problem). These two essential characteristics act opposite to each other, and therefore deep knowledge of their effects on the positioning and dynamic behaviour of investigated systems is important in design. An important task is to find the optimal combination of friction and clearances related to desired dynamic characteristics of a mechatronic device. An attempt to evaluate the interaction of various clearances was introduced in [3]. The main goal of our contribution is to qualitatively analyze mutual interaction of joint clearances and friction effects on the dynamic behaviour of chosen multibody systems.

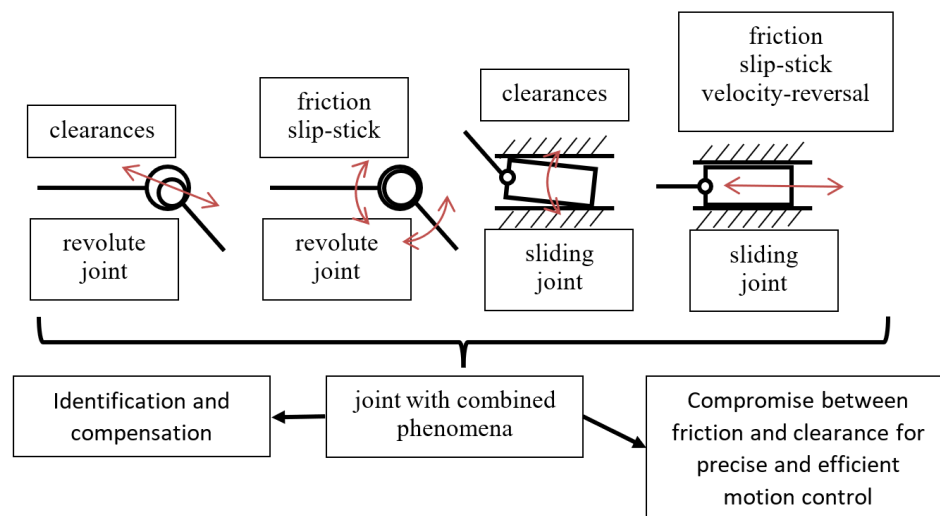


Figure 1: Scheme of the whole research methodology about the interaction of clearances and friction

### 2 Modelling

The very first step of the methodology is the formulation of a proper computational model. Considering various selection of the type of coordinates describing the system of coupled rigid bodies we can obtain various types of mathematical equations. Cartesian (physical) coordinates are the obvious choice in the case of this paper with the purpose to characterize imperfect joints. The position of each rigid body in the 3D space can be determined for by six coordinates (three displacements and three rotation). The well-known mathematical model based on the Lagrange equations can be expressed as the set of differential-algebraic equations of index one in the form

$$\begin{bmatrix} \mathbf{M} & \Phi_q^T \\ \Phi_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ -\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) \\ \boldsymbol{\gamma}(\mathbf{q}, \dot{\mathbf{q}}, t) \end{bmatrix}. \quad (1)$$

Matrix  $\mathbf{M}$  is the global mass matrix of the multibody system, vector  $\mathbf{q}$  is the vector of generalized coordinates of all bodies and vector  $\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$  contains centrifugal and Coriolis inertia forces, elastic and damping forces and other externally applied forces including the gravity.

This form of equations of motion was obtained after double differentiation of constraint equations  $\Phi = \mathbf{0}$  with respect to time, which describe ideal joints. Vector of Lagrange multipliers  $\lambda$  is introduced in equation (1). Vector  $\boldsymbol{\gamma}(\mathbf{q}, \dot{\mathbf{q}}, t)$  represents the remaining terms after the constraints differentiation. Generally, effects of contacts and impacts can be included in vector  $\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$ . Therefore imperfect joint models and friction present in real joints are naturally incorporated into vector  $\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t)$ . Various ways of modelling imperfect joints in the framework of multibody system dynamics was summarized in [4]. Analogously, the survey of friction models and their mathematical description was introduced e.g. in [5]. The implemented software contains both static and dynamic friction models in order to have a possibility of model comparison.

Solution of equations of motion (1) can be based e.g. on elimination of Lagrange multipliers and further direct integration of the underlying ordinary differential equation. The presented modelling methodology was implemented by the authors into in-house software in the MATLAB environment.

### 3 Qualitative dynamic analysis and conclusions

Several typical mechanical systems like a slider crank mechanism, a four-bar mechanism and their spatial modifications were modelled by means of the implemented software. The applications were limited to rigid body systems with rotational, translational and spherical joints (both ideal and real with clearances) and passive effects. The equations of motion of these systems were solved using numerical integration. A certain range of operating conditions was considered in all analyses and chosen kinematical quantities were evaluated in order to explore the mutual interaction of joint clearances and friction effects on system dynamics. It can be concluded that for various combinations of joint clearances and friction levels, the dynamic behaviour differs qualitatively. Nonlinear dynamic behaviour of the system obtained by numerical simulation is evaluated based on typical analytical approaches [6].

The future goal is to use gained knowledge in order to proposed suitable compensation mechanism of both mutually interacting effects. It was shown how to reduce standalone undesirable friction effects in [7], however, we plan to combine clearances and friction together as in the real machines including an experimental validation.

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