Parameter identification method for multibody systems incorporating the adjoint method and the proper orthogonal decomposition

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EXTENDED ABSTRACT

1 Introduction

Many mechanical systems such as vehicles, robots and space structures are comprised of a lot of interconnected components. Even though the multibody dynamics is one of effective methods to analyze nonlinear dynamic behavior for such complex mechanical systems, it is quite difficult to obtain all parameters required for numerical simulation by only direct measurements. Therefore, parameter estimation and identification techniques are important for application of the multibody dynamics to practical problems [1]. This study aims to develop a parameter identification technique based on the adjoint method [2, 3]. Moreover, in order to evaluate effects of model uncertainties on an accuracy of the parameter identification, this study introduces the proper orthogonal decomposition (POD) to a cost function used in the adjoint method. The present method is applied to a parameter identification for a two-dimensional beam with elastic supports. Then, the validity of the present method is discussed by numerical experiments.

2 Method

In order to consider model uncertainties at support ends, this study considers a flexible beam supported by springs and dampers at its both ends as shown in Fig. 1. Here, we employ the floating frame of reference formulation for the description of the system. The equations of motion can be expressed by the differential algebraic equations (DAEs) as

$$\dot{\boldsymbol{q}} = \boldsymbol{v}, \quad [\mathbf{M}]\dot{\boldsymbol{v}} + \frac{\partial \boldsymbol{C}^T}{\partial \boldsymbol{q}}\boldsymbol{\lambda} = \boldsymbol{F}^{(e)}, \quad \boldsymbol{C}(\boldsymbol{q}) = \boldsymbol{0}. \tag{1}$$

The parameter identification can be achieved by finding unknown parameters $\boldsymbol{p}$ minimizing a cost function J given by

$$J = \int_0^{T_s} L\,dt, \qquad L = \frac{1}{2}[\boldsymbol{s}(t) - \bar{\boldsymbol{s}}(t)]^T[\boldsymbol{s}(t) - \bar{\boldsymbol{s}}(t)], \tag{2}$$

where $\bar{\boldsymbol{s}}$ denotes the measured data and $\boldsymbol{s}$ represents the system outputs. In this study, the state variables $\boldsymbol{x}$ and $\boldsymbol{v}$ calculated by Eq. (1) are used as the system outputs $\boldsymbol{s}$. In an application of the adjoint method to the multibody system analysis [2], the cost function in Eq. (2) is enhanced by incorporating the DAEs in Eq. (1) as follows:

$$\hat{J} = J + \int_0^{T_s} \boldsymbol{\xi}^T(\dot{\boldsymbol{q}} - \boldsymbol{v})dt + \int_0^{T_s} \boldsymbol{\zeta}^T\left([\mathbf{M}]\dot{\boldsymbol{v}} + \frac{\partial \boldsymbol{C}^T}{\partial \boldsymbol{q}}\boldsymbol{\lambda} - \boldsymbol{F}^{(e)}\right)dt + \int_0^{T_s} \boldsymbol{\mu}^T\boldsymbol{C}dt, \tag{3}$$

where $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$ represent the adjoint variables corresponding to each equation in the DAEs (1). The adjoint variables can be obtained from computation for the adjoint equations derived by the variation of the cost function $\hat{J}$. And then, if $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$ satisfying the adjoint equations are obtained, the gradient $\nabla_{\boldsymbol{p}}\hat{J}$ with respect to the unknown parameter $\boldsymbol{p}$ can be calculated.

In addition, the present method applies the POD to a set of measured data $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}$. These variable can be expressed by using the proper orthogonal decomposition modes (POMs) as follows:

$$\bar{\boldsymbol{u}}(t) = \sum_{i=1}^{N_d} \bar{a}_i(t)\bar{\boldsymbol{w}}_i, \qquad \boldsymbol{u}(t) = \sum_{i=1}^{N_d} a_i(t)\boldsymbol{w}_i, \tag{4}$$

where $\bar{\boldsymbol{w}}_i$ and $\boldsymbol{w}_i$ denote the POMs obtained from $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}$, respectively. Since $\bar{a}_i$ and a_i can be calculated by $\bar{a}_i = \bar{\boldsymbol{u}}^T\bar{\boldsymbol{w}}_i$ and $a_i = \boldsymbol{u}^T\boldsymbol{w}_i$, the cost function with the POD can be defined as

$$J_i^{\mathrm{POD}} = \int_0^{T_s} L_i^{\mathrm{POD}}\,dt, \qquad L_i^{\mathrm{POD}} = \frac{1}{2}(a_i(t) - \bar{a}_i(t))^2. \tag{5}$$

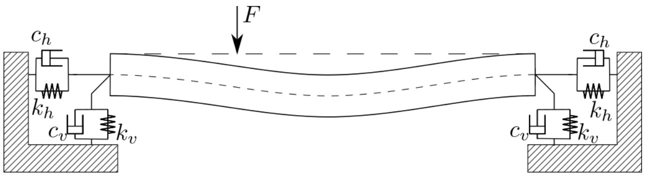

Figure 1: Flexible beam supported by springs and dampers

Table 1: Parameters for simulation

Spring constant	$^*k_h$	[N/m]	5000
	$^*k_v$	[N/m]	1000
Damping coefficient	$^*c_h$	[Ns/m]	50
	$^*c_v$	[Ns/m]	50
Impulse force	F	[N]	100
Number of elements	N		10

3 Results

The present method is validated by a parameter identification test for the flexural rigidity EI. Parameters of a numerical simulation for the parameter identification test are listed in Table 1. In this table, the values with asterisk (*) are treated as uncertainties. A true value for the flexural rigidity is $EI = 500$ Pa. Figure 2 represents time history of beam displacements in the vertical direction which is used for calculating the POMs. Then, results of the parameter identification are summarized in Table 2. It can be found that the cost functions given by the 1st and 2nd POMs fail to estimate the true value, even though such POMs have higher contribution ratios. On the other hand, the estimated values from the cost function given by more than the 4th POMs show in good agreement with the true value.

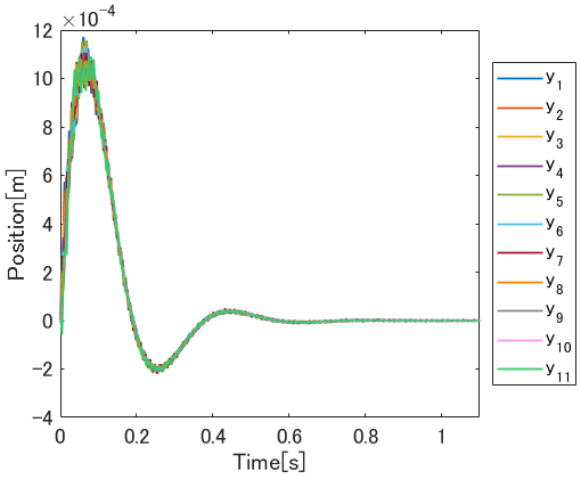

Figure 2: Time history of vertical displacement

Table 2: Results of parameter identification for EI calculated by J_i^{POD} (true value: 500 Pa)

POM i	1	2	3	4	5	6	7	8	9	10	11
EI	–	–	502	500	500	500	500	500	500	500	500

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