Adaptive step size determination for convex optimization

Ekansh Chaturvedi[1], Jyotirmoy Mukherjee[2], Corina Sandu[3]

[1]Dept. of Mechanical Engineering
Virginia Tech
Blacksburg, VA 24061
ekanshchat96@vt.edu

[2]Dept. of Mechanical Engineering
Virginia Tech
Blacksburg, VA 24061
jmukherjee@vt.edu

[3]Dept. of Mechanical Engineering
Virginia Tech
Blacksburg, VA 24061
csandu@vt.edu

EXTENDED ABSTRACT

1 Introduction

Convex optimization problems are crucial to several applications like design optimization and optimal control. The objective of such an optimization problem is to find an optimizer in the domain of the design variables which either minimizes or maximizes a certain cost function. The solution process generally involves determining a search direction and step length to advance in the search direction such that, with each new step, the cost reaches closer to the optimal value. The step length plays a key role as it directly affects computational time. A fixed step length that is very small could lead to an optimal solution, however at a large computational cost. A larger step size could accelerate the process however, it could result in overshoot and miss the optimum that is queried.

Several approaches are available in literature to determine the optimum value of the step length that could improve computational efficiency and not compromise on attaining an optimum value if there exists one. For example, methods like interval halving, Golden search method, brackets the design variable domain set if the optimum value lies within the bracketed set. Besides fixed step size search, methods such as accelerated step size search, quadratic interpolation method can be used to determine the optimum step length. However, it is seen that the available algorithms have both advantages and disadvantages. For instance, it is observed that certain algorithms could be very slow to converge while others may not converge at all [1]. In the case when the optimum cannot be found in an initial interval of uncertainty, methods like the Golden Section and Fibonacci Search are used. It has also been observed that it could benefit to combine certain algorithms to determine the optimal step size. According to [1], in the absence of first derivatives, quasi-Newton approach is extremely efficient while Newton method work efficiently if first and second derivatives exist. Although, several state-of-the-art approaches exist in solving the optimization problem for convex functions, the need for faster computation is crucial to find real time applications, such as online Model Predictive Control (MPC) in robotic platforms that have limited onboard computational power. In this work, a novel adaptive step size determination algorithm is proposed and investigated. The optimal value of the step size is determined by considering the trend of the gradient norm of the convex function and the absolute value of the cost.

2 Problem Statement

Let $\mathbf{x} \in \mathbb{R}^n$ be a vector of optimization variables and $f_O: D \rightarrow \mathbb{R}$ be a scalar valued function that is convex. The constrained minimization problem is posed as:

$$\min f_O(\mathbf{x}) \qquad st.\, g(\mathbf{x}) \le 0; h(\mathbf{x}) = 0 \tag{1}$$

3 Methodology

The unconstrained convex optimization problem is solved iteratively with an initial guess of the design variable $\mathbf{x}_0 \in \mathbb{R}^n$ and consequently determining the next design point $\mathbf{x}_k \in \mathbb{R}^n$ such that $f(\mathbf{x}_o) < f(\mathbf{x}_k)$. This is obtained by iteratively calculating for the next design point, $\mathbf{x}_{k+1} = \mathbf{x}_k + \lambda \mathbf{s}$ where, λ is the step size and $\mathbf{s}$ is the search direction. Since the problem is convex, it is known that the local minimum is the global minimum. Further, it can be observed that the closer the design variable reaches the optimum, both the cost function as a function of the design variable as well as the norm of the gradient at consecutive design variables become smaller and smaller. Hence, if $\mathbf{x}_k$ and $\mathbf{x}_{k+1}$ are the consecutive design points in the search space D, the necessary and sufficient conditions are:

$$f(\mathbf{x}_{k+1}) < f(\mathbf{x}_k) \qquad \equiv \qquad \left\| \frac{f(\mathbf{x}_{k+1})}{f(\mathbf{x}_k)} \right\| < 1 \tag{2}$$

$$\nabla f(\mathbf{x}_{k+1}) \le \nabla f(\mathbf{x}_k) \qquad \equiv \qquad \left\| \frac{\nabla f(\mathbf{x}_{k+1})}{\nabla f(\mathbf{x}_k)} \right\| \le 1 \tag{3}$$

The adaptive step size scheme is used iteratively until the norms of the ratios of gradient and cost reach below a tolerance limit.

4 Preliminary Results

The proposed adaptive step size determination scheme is implemented in a case study problem [3] posed as follows. The feasible region is shown as follows in Figure 1. The problem was solved using sequential quadratic programming (SQP). The proposed adaptive step size algorithm was tested against the renowned Golden section method. The key computational characteristics are compared in the Table 1.

$$Min \qquad f(A,h) = 0.6h + 0.001A \tag{4}$$

$$st. \qquad h \leq 21; \quad Ah \geq 70000; \quad A(h+14) \leq 140{,}000; \quad A > 0; h > 0$$

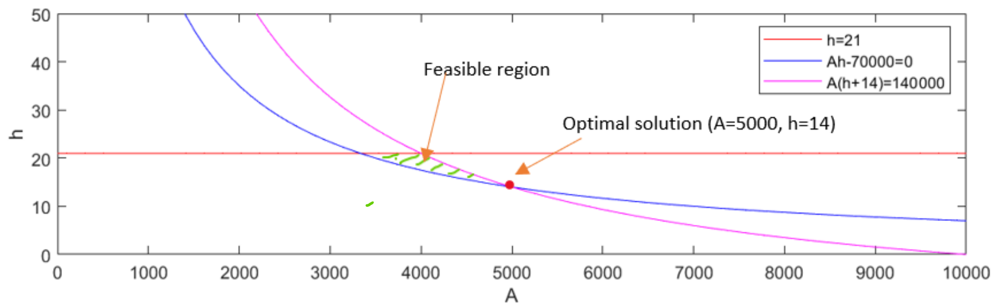

Figure 1: Feasible region within constraint lines

Table 1: Computational metrics

Metric	Golden section method	Proposed algorithm
Number of iterations	6	3
Computing time (sec)	0.56	0.36

The algorithm to solve the optimization problem was programmed on Matlab R2022. The optimum value was obtained in 0.36 seconds using the adaptive step size determination scheme and the solution was observed to converge in 3 iterations. The same problem was solved using the Golden Section method to get step length; and the same optimum values were obtained, however, it was observed that the Golden search approach took 6 iterations and 0.56 seconds.

4 Application to Multibody Dynamic Systems

Design optimization is an integral part of machine design and hence, in multibody dynamic system. The computational cost associated with such algorithms is an important characteristic for processing efficiency. The proposed optimization algorithm can be directly applied to generic formulations of multibody systems. In this study, the proposed algorithm will be implemented on a multibody dynamics system design optimization problem. The results will be compared against the existing methods and the inferences will be discussed.

5 Conclusions

A new method to obtain the step size in convex optimization problem is proposed. The proposed method was observed to converge faster and with lower number of iterations against standard solvers for a Sequential Quadratic Program case study. A standard approach to validate the applicability of optimization algorithms is to test against benchmark problems. Several such problems are found in literature such as the Auckley function, Rosenbrock function, Beale function, Rastrigin function, Levi function, Bukin function, Three camel function etc. Future work includes testing the developed adaptive step size scheme using Rosenbrock function.

References

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