# Numerical Optimization of Parameters Using the Covariance Matrix Adaptation Evolution Strategy in the Coupled Multibody Systems

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## EXTENDED ABSTRACT

### 1 Introduction

The coupled multibody and hydraulics modeling methods [1] are often used in the real-time simulation and analysis of hydraulically driven machines such as excavators, wheel loaders, and tractors. Through real-time simulation methods, the human operator commands can interact with the simulated machines via simulators or hardware replicating the real machine operation. In the coupled modeling methods [1], the lumped fluid theory [2] computes the hydraulic pressure derivatives by dividing the effective bulk modulus with small volumes. However, the lumped fluid theory introduces numerical stiffness in the coupled simulation methods affecting the computational efficiency. This problem gets more profound in the simulation of complicated industrial systems. To this end, to optimize the system states, this study proposes the use of a simplified hydraulic model with the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [3] as an alternative approach to the lumped fluid theory. The CMA-ES is preferred over other optimization algorithms due to the requirement of less number of generations, better search properties, and ease of using parallelization methods [3]. As an example, the simplified hydraulic model with CMA-ES is implemented on a hydraulically actuated four bar mechanism. The results of the proposed approach are compared with the lumped fluid theory. This new approach could be used to improve the computational efficiency of the coupled multibody simulation methods.

#### 2 Modeling of Coupled Systems

**Multibody systems** A mechanical closed-loop system can be expressed into the independent position vector  $\mathbf{z}^{i}$  and velocity vector  $\mathbf{\dot{z}}^{i}$  using the coordinate partitioning method as,

$$\begin{bmatrix} \dot{\mathbf{z}}^{i} \\ \ddot{\mathbf{z}}^{i} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{z}}^{i} \\ (\mathbf{M}^{\Sigma})^{-1} \mathbf{Q}^{\Sigma} \end{bmatrix} \equiv \mathbf{f}(\mathbf{x}),$$
(1)

where  $\mathbf{x} = \begin{bmatrix} \mathbf{z}^i & \dot{\mathbf{z}}^i \end{bmatrix}^T$ ,  $\mathbf{M}^{\Sigma} = \mathbf{R}_z^T \mathbf{R}_d^T \mathbf{T}^T \overline{\mathbf{M}} \mathbf{T} \mathbf{R}_d \mathbf{R}_z$  and  $\mathbf{Q}^{\Sigma} = \mathbf{R}_z^T \mathbf{R}_d^T (\mathbf{T}^T \overline{\mathbf{Q}} - \mathbf{T}^T \overline{\mathbf{M}} \mathbf{D})$  represent the accumulated mass matrix and accumulated force vector, respectively. Here,  $\mathbf{R}_z$  is the velocity transformation matrix,  $\mathbf{R}_d$  is the block-diagonal matrix,  $\mathbf{T}$  is the constant path matrix,  $\overline{\mathbf{M}}$  is the composite mass matrix of the system,  $\overline{\mathbf{Q}}$  is vector of the composite forces and  $\mathbf{D}$  represent the absolute accelerations, when the vector of accelerations  $\mathbf{\ddot{z}}$  is zero. The reader is referred to [1] for the details of the coordinate partitioning method. The force produced by the hydraulic cylinder  $F_h = p_1 A_1 - p_2 A_2 - F_{\mu}$  is combined with  $\overline{\mathbf{Q}}$  to model the hydraulically driven systems. Here,  $p_1$ ,  $p_2$ ,  $A_1$  and  $A_2$  are the pressures and areas on the piston and piston-rod side, respectively.  $F_{\mu}$  represents the friction force.

**Lumped Fluid Theory** The pressures and the effective bulk modulus  $B_{e_h}$  in the hydraulic circuit can be modelled using the lumped theory [2] as,

$$\dot{p}_{h} = \frac{B_{e_{h}}}{V_{h}} \left( -\frac{dV_{h}}{dt} + Q_{s} \right), \quad B_{e_{h}} = \left( \frac{1}{B_{o}} + \sum_{c=1}^{n_{h}} \frac{V_{c}}{V_{h}B_{c}} \right)^{-1}, \tag{2}$$

where  $\dot{p}_h$  is the first derivative of hydraulic pressure,  $V_h$  is the hydraulic volume, and  $Q_s$  is the sum of incoming and outgoing flows. Moreover,  $B_o$  is the oil bulk modulus,  $V_c$  is the sub-volume, and  $B_c$  is the bulk modulus of the sub-volume. The flow rate in Eq. (2) can be computed using the semi-empirical method [2]. For instance, using this method, the flow rate  $Q_d$  through a directional control valve can be expressed as  $Q_d = C_v U \operatorname{sgn}(\Delta p) \sqrt{|\Delta p|}$ .  $C_v$  is the semi-empirical flow rate coefficient, U is the relative position of the spool,  $\operatorname{sgn}(\cdot)$  is the signum function to define the direction of flow rate, and  $\Delta p$  is the pressure difference over the valve ports. The voltage signal U can be expressed in differential form as  $\dot{U} = \frac{U_{ref} - U}{\tau}$ . Here,  $U_{ref}$  defines the reference voltage signal and  $\tau$  is the time constant describing the valve dynamics.

**Simplified hydraulics model** The hydraulic force  $F_s$  can be expressed using the simplified hydraulics model as,

$$F_s = aU_i + b_0 + b_1 s_0 + b_1 s_i + b_2 \dot{s}_i, \tag{3}$$

where *a* is the hydraulic force gain parameter, and  $b_0$ ,  $b_1$  and  $b_2$  are the hydraulic force bias parameters.  $U_i$  is the input signal,  $s_i$  is the actuator position,  $\dot{s}_i$  is the actuator velocity at the time step *i* and  $s_0$  is the initial actuator position.

#### **3** Results and Conclusion

Figure 1 shows a hydraulically actuated four bar mechanism. The mechanism is actuated by a constant pressure source  $p_P$  via a 4/3 directional control valve and a tank of pressure  $p_T$ . Angle  $z_1$  and angular velocity  $\dot{z}_1$  of the body 1 in the mechanism are optimized using the proposed approach.



Figure 1: Hydraulically actuated four bar mechanism.  $Q_{d1}$  and  $Q_{d2}$  are the flow rates in the control volumes in  $V_1$  and  $V_2$ .

**Computational accuracy and efficiency** Figures 2(a)–2(c) describe the numerical results of CMA-ES application against the lumped fluid theory. In the Figures, the light grey color region demonstrates the extension, and the light orange region represents the retraction of the hydraulic cylinder. As can be seen, the CMA-ES algorithm enables the computation of  $z_1$  and  $\dot{z}_1$ . Further, the percent normalized root mean square errors in simplified hydraulics with respect to lumped theory are 0.26 % and 0.23 % for  $z_1$  and  $\dot{z}_1$ , respectively. This error can be further reduced by tuning the parameters of CMA-ES and simplified hydraulics. The computational efficiency of simplified hydraulics with CMA-ES against the lumped fluid theory is shown in Figure 2(c) during the simulation time. In the MATLAB environment, the CMA-ES driven approach is approximately 10 times faster than the lumped fluid theory.



Figure 2: Numerical accuracy and computational time of simplified hydraulics against the lumped fluid theory.

This approach demonstrates an alternative approach to the lumped fluid theory improving the computational efficiency of the coupled multibody simulation methods. In future studies, this approach can be coupled with the multibody system dynamics formulation to explain its computational accuracy and efficiency benefits. Further, the CMA-ES approach could be used with the state and parameter estimation, artificial intelligence, and deep learning methods to optimize system states and parameters as it improves the accuracy and reduces the noise in sensors measurements [3].

#### References

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