A new approach to formulate structural methods for multibody dynamics

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EXTENDED ABSTRACT

1 Introduction

The use of the so called structural integrators is becoming a mainstream in Multibody Dynamics [1], [2], [3]. These methods allow one to directly integrate the Differential Algebraic Equations (DAEs) that appear in Multibody Dynamics and other phenomena without the need of reducing the DAE index, although this can lead in the case of implicit methods to difficult to predict stability issues, which can be avoided by integrating in minimal coordinates.

Currently the most used implicit structural integrators in multibody dynamics are the Newmark method and the HHT method. Recently, the second order central difference method (SOCDM) has also succesfully being introduced. This method is considered as explicit for its use in structural dynamics, but in the case of 3D Multibody Dynamics it becomes implicit due to the dependency on the velocity that appears in the differential equation. Probably the biggest drawback of these methods is that they are limited to order 2 convergence. The classical formulation of SOCDM used in structural dynamics leads also to a cancellation problem that reduces its precision.

In this document the authors will present a reformulation of both the Newmark and the SOCDM methods, which allow one to overcome the problems of cancellation in the classical SOCDM and leads to a configurable and extensible formulation of the methods. The extension of these reformulations leads to integrators which exhibit higher convergence than their classical counterpart (obviously at the cost of reduction of the stability conditions).

2 Classical formulations for Newmark and SOCDM

The central differences method is based on the following central differences expressions:

$$\dot{\boldsymbol{x}}(t)=\frac{\boldsymbol{x}(t+\Delta t)-\boldsymbol{x}(t-\Delta t)}{2\Delta t} \quad , \quad \ddot{\boldsymbol{x}}(t)=\frac{\boldsymbol{x}(t+\Delta t)-2\boldsymbol{x}(t)+\boldsymbol{x}(t-\Delta t)}{\Delta t^2} \tag{1}$$

which, along with the equilibrium equation evaluated in the beginning of the timestep:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}(t)\dot{\boldsymbol{x}}(t)+\boldsymbol{K}(t)\boldsymbol{x}(t)=\boldsymbol{f}(t) \tag{2}$$

conform the integrator. This integrator allow one to obtain $\boldsymbol{x}(t+\Delta t)$, $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ from $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t-\Delta t)$ and $\ddot{\boldsymbol{x}}(t-\Delta t)$. For structural dynamics this is usually reformulated leading to a multipoint formulation [3].

The Newmark method is based instead in the following equations:

$$x(t+\Delta t)=x(t)+\Delta t\cdot\dot{x}(t)+\frac{(\Delta t)^2}{2}\left((1-\alpha_0)\ddot{x}(t)+\alpha_0\ddot{x}(t+\Delta t)\right) \tag{3}$$

$$\dot{x}(t+\Delta t)=\dot{x}(t)+\Delta t\left((1-\alpha_1)\ddot{x}(t)+\alpha_1\ddot{x}(t+\Delta t)\right) \tag{4}$$

In this case, the equilibrium equation is formulated in $t+\Delta t$:

$$\boldsymbol{M}(t+\Delta t)\ddot{\boldsymbol{x}}(t+\Delta t)+\boldsymbol{C}(t+\Delta t)\dot{\boldsymbol{x}}(t+\Delta t)+\boldsymbol{K}(t+\Delta t)\boldsymbol{x}(t+\Delta t)=\boldsymbol{f}(t+\Delta t) \tag{5}$$

It is easy to find out that the SOCDM can be rewritten in a similar form to that of Newmark by using the following equations:

$$x(t+\Delta t)=x(t)+\Delta t\,\dot{x}(t)+\frac{\Delta t^2}{2}\ddot{x}(t) \quad , \quad x(t-\Delta t)=x(t)-\Delta t\,\dot{x}(t)+\frac{\Delta t^2}{2}\ddot{x}(t) \tag{6}$$

Which can be considered a particular case of the set:

$$x(t+\Delta t)=x(t)+\Delta t\,\dot{x}(t)+\frac{\Delta t^2}{2}\left(\alpha_0\ddot{x}(t)+(1-\alpha_0)\ddot{x}(t-\Delta t)\right) \tag{7}$$

$$\dot{x}(t)=\dot{x}(t-\Delta t)+\Delta t\left(\alpha_1\ddot{x}(t)+(1-\alpha_1)\ddot{x}(t-\Delta t)\right) \tag{8}$$

The introduction of the parameters α_0 and α_1 allow for a configurable method, as in the case of Newmark. As equations (6) are a Taylor expansion, one can extend them to a higher order:

$$x(t+\Delta t)=x(t)+\Delta t\dot{x}(t)+\frac{\Delta t^2}{2}\ddot{x}(t)+\frac{\Delta t^3}{6}\left((1-\alpha)\dddot{x}(t-\Delta t)+\alpha\dddot{x}(t)\right) \quad (9)$$

$$\dot{x}(t)=\dot{x}(t-\Delta t)+\Delta t\ddot{x}(t-\Delta t)+\frac{\Delta t^2}{2}\left((1-\beta)\dddot{x}(t-\Delta t)+\beta\dddot{x}(t)\right) \quad (10)$$

$$\ddot{x}(t)=\ddot{x}(t-\Delta t)+\Delta t\left((1-\gamma)\dddot{x}(t-\Delta t)+\gamma\dddot{x}(t)\right) \quad (11)$$

Which leads to method of higher convergence. A similar approach can be done with Newmark, thus allowing one to extend those methods. One can even take more Taylor series elements for more convergence. The use of this configuration also leads to the ellimination of the aformentioned cancellation issues.

3 Results

The simple pendulum presented in the IFTOMM Multibody Benchmark will be addressed.

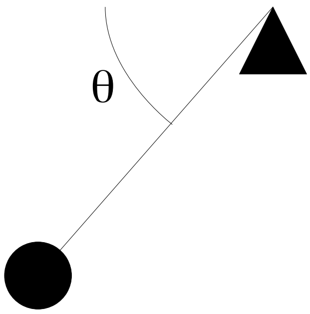

Figure 1: Planar simple pendulum

The pendulum moves without external forces and thus its mechanical energy must be conserved, and it is used as an indication of precision.

Table 1. Planar simple pendulum. Obtained results

Timestep	Error, Central Differences	Error, Reformulated CD	Error, higher order method
1e-3	2.00492e-05	2.00492e-05	8.67265e-7
1e-4	2.00597e-07	2.00492e-7	8.67649e-10
1e-5	2.46675e-08	2.01399e-09	1.88095e-11
1e-6	3.66886e-07	2.26645e-11	2.48228e-11

3 Conclusions and future work

An approach for the generalization of the structural methods has been presented. It allows one to develop methods with a higher convergence order and a better numerical behavior. Although here it has not been presented, this comes at the cost of the range of stability of the method, but is of great interest in problems of low stiffness.

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