Modified Sliding Mode Control of Redundantly Actuated Parallel Kinematic Mechanisms with Uncertain Model

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EXTENDED ABSTRACT

1 Introduction

The model of a multibody system is never fully precise. However, it can be detected only by redundant measurement, i.e. by measurement of more variables than the number of DOFs is. It is naturally present at multibody systems with parallel kinematic structure that are redundantly actuated. Redundant actuation means that the number of drives is larger than the number of DOFs. Each drive is equipped with its own measurement system and thus the number of measurement variables is larger than the number of DOFs.

Recently, it was derived and demonstrated at many papers, e.g. [1], that any control law for redundantly actuated parallel kinematic mechanisms (PKM) where control action is derived from differences between real and desired motion has principal problems. The principal problem is that some part of resulting error dynamics cannot be influenced and eliminated by increase of control gains and even the resulting error dynamics could become unstable. Certainly, such problems are proportional to the deviation of model from real machine and they are not manifested for small deviations. A solution by robust Sliding Mode Control (SMC) [2] was proposed in [3]. But the complete solution was only found in [4] that required the modification of robust SMC. This paper describes this modification of robust SMC and its application for control of redundantly actuated PKM.

2 Uncertain kinematic and dynamic model of PKM

A multibody model of the redundantly actuated PKM can be described by n general coordinates $\mathbf{q}$. PKM is characterized by closed kinematic loops leading to $r<n$ geometric constraints

$$\mathbf{g}(\mathbf{q}, \mathbf{l}) = \mathbf{0} \tag{1}$$

where $\mathbf{l}$ is a vector of a geometric parameters (model). The geometric parameters $\mathbf{l}$ differ from the real parameters $\underline{\mathbf{l}}$ of the real mechanisms. It is assumed that the geometric parameters are known with bounded deviations $|\Delta\mathbf{l}| \leq \Delta\mathbf{l}_{\max}$, ($\underline{\mathbf{l}}=\mathbf{l}+\Delta\mathbf{l}$). The dynamics of the redundantly actuated PKM is given by the equations of motion after transformation to independent coordinates

$$\ddot{\mathbf{q}}_2 = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{l}, \mathbf{m}) + \mathbf{B}(\mathbf{q}, \mathbf{l}, \mathbf{m})\mathbf{u} \tag{2}$$

where $\mathbf{q}_2$ is a vector of independent coordinates, $\mathbf{m}$ is a vector of a dynamic parameters and $\mathbf{u}$ is a vector of the control inputs. For the real mechanism the dynamic parameters $\mathbf{m}$ differ from the real parameters $\underline{\mathbf{m}}$. It is again assumed that the dynamic parameters are known with bounded deviations $|\Delta\mathbf{m}| \leq \Delta\mathbf{m}_{\max}$, ($\underline{\mathbf{m}}=\mathbf{m}+\Delta\mathbf{m}$).

3 Control problem of redundantly actuated PKM

The dynamic model (2) accompanied by PD control law with elimination of mutual fighting of redundant actuators finally leads to the dynamic model of control errors $\mathbf{e}_2$ [1], [4]

$$\overline{\mathbf{G}}\ddot{\mathbf{e}}_2 + \overline{\mathbf{C}}\dot{\mathbf{e}}_2 + \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}}_2 + \mathbf{K}_{\mathrm{P}}\boldsymbol{e}_2 + \Delta\overline{\mathbf{G}}\ddot{\mathbf{q}}_2 + \Delta\overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \Delta\overline{\mathbf{Q}} - \mathbf{S}\left(\overline{\mathbf{G}}\ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2^{d} + \overline{\mathbf{Q}}\right) - \Delta\mathbf{A}^{T}\mathbf{N}_{\mathbf{A}^{T}}\mathbf{u}_0 = \mathbf{0} \tag{3}$$

where $\mathbf{e}_2$ is a vector of control errors, $\mathbf{G}$ is a generalized mass matrix, $\mathbf{C}\dot{\boldsymbol{q}}_2$ represents generalized Coriolis and centrifugal forces, $\mathbf{Q}$ contains all remaining impressed forces (potential, friction,...), $\mathbf{A}^{\mathrm{T}}$ is a control matrix, $\mathbf{u}$ is a vector of control forces, $\mathbf{K}_{\mathrm{P}}$, $\mathbf{K}_{\mathrm{D}}$ proportional and derivative control gains, $\mathbf{S}$ matrix of parasitic feedback, $\mathbf{N}_{\mathrm{AT}}$ is orthogonal complement of control matrix $\mathbf{A}^{\mathrm{T}}$, $\mathbf{u}_0$ is a vector of control inputs invariant with respect to control matrix $\mathbf{A}^{\mathrm{T}}$, Δ means dynamics deviation due to uncertain model, indexes d mean desired values. It is evident that the control gains $\mathbf{K}_{\mathrm{P}}$, $\mathbf{K}_{\mathrm{D}}$ can influence the behavior of only first four terms in (3) and the remaining terms cannot be influenced by control gains at all. And this is the principal problem resulting error dynamics cannot be influenced and eliminated by increase of control gains and even the resulting error dynamics could become unstable.

4 Modified robust SMC and its application

SMC is a control that can eliminate the uncertain dynamics if the deviations have known limits [2]. SMC is applicable to dynamic systems with equal number of inputs and blocks in Brunovsky canonical form, i.e. after exact input-state feedback linearization with zero zero dynamics. It works with dynamic Brunovsky canonical blocks

$$x^{(n_j)} = \mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x}) + (\mathbf{b}(\mathbf{x}) + \Delta\mathbf{b}(\mathbf{x}))\mathbf{p} \tag{4}$$

where **x** are state variables of Brunovsky canonical blocks of order n_j, **f** and **b** are standard control dynamics descriptions, Δ are the bunded dynamics deviations and **p** is the input (control) variable. For redundantly actuated systems the number of control variables **u** is larger than the number of Brunovsky canonical blocks and the input variables **p** are auxiliary inputs that are solved from the equation derived by comparison of equations (2) and (4)

$$\mathbf{Bu} = \mathbf{Dp} \tag{5}$$

where **D**=diag(b_j). In case of the redundantly actuated PKM there is no unique solution for the real control inputs **u** as the matrix **B** is a rectangular one. The ambiguity of solution can be used for fulfillment of the secondary control tasks (anti-backlash, active stiffness control, etc.). In order to apply the robust version of SMC with guaranteed convergence and stability [2] it is necessary to modify the uncertainty of dynamic equation (4) by the dynamic equation [4]

$$x^{(n_j)} = \mathbf{f}(\mathbf{x}) + (\Delta\mathbf{f}(\mathbf{x}) + \Delta\mathbf{b}(\mathbf{x}))\mathbf{p}) + \mathbf{b}(\mathbf{x})\mathbf{p} \tag{6}$$

for which the known limit of dynamics deviation can be computed

$$|\Delta\mathbf{f}(\mathbf{x}) + \Delta\mathbf{b}(\mathbf{x}))\mathbf{p}| \leq \Delta\mathbf{f}_{max} + \Delta\mathbf{b}_{max}\mathbf{p}_{max} \tag{7}$$

where index *max* means maximum bounded values of particular terms. Then the algorithm of robust SMC from [2] for the redundantly actuated dynamics by the solution of equation (5) with resulting guaranteed convergence and stability can be computed.

An example of planar redundantly actuated PKM called Crosshead (Fig. 1a) is illustrated by the resulting dynamics of platform angle φ in Fig. 1b. Important fact is visible in Fig. 1b that the dynamics converges to the values of the model not to the values of the unknown real mechanism of PKM.

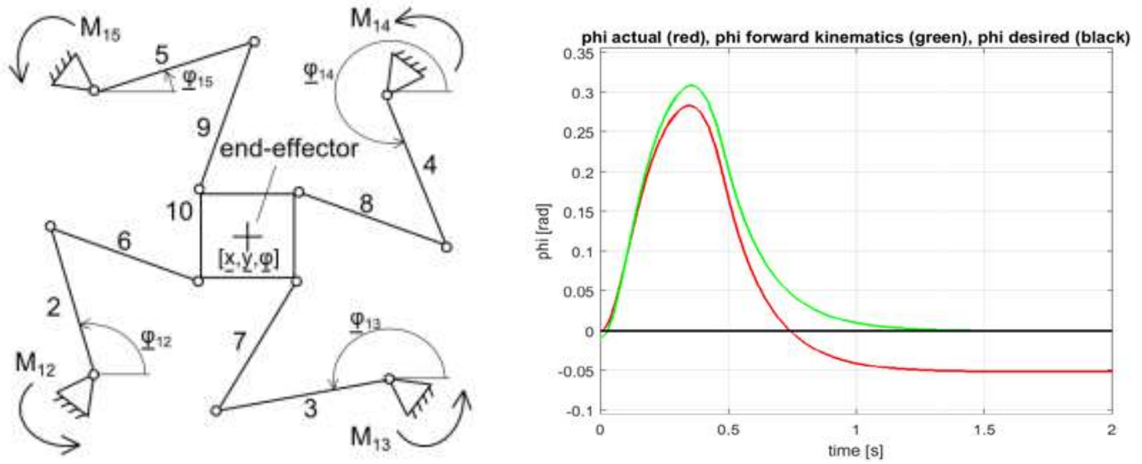

Figure 1: (a) PKM Crosshead (b) Time behavior of platform angle φ

5 Conclusions

The paper describes the solution of serious control problem of redundantly actuated PKM [1] and thus it reopens the way for efficient control of redundantly actuated PKM. This enables to use all benefits of redundantly actuated PKMs.

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References

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