A Computational Framework for Optimal Locomotion in Limbless Organism

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EXTENDED ABSTRACT

1 Introduction

Limbless organisms perform a wide range of locomotion patterns viz undulatory (C. elegans), crawling (maggots), or inching (caterpillar) gait, to name a few. Although one-dimensional beam-based element has been developed to simulate these patterns, still the origin of specific locomotion patterns is not fully understood [1]. Furthermore, a 3D mechanics-based model considering the active response of muscle activity and anisotropic interface frictional forces is not available in the literature.

We propose a general 3D finite element-based computational framework to study the locomotion pattern in the limbless organism. In particular, a novel contact law and growth model is developed to characterise the distinct gait performed by limbless worms on the soft substrate. Additionally, optimal locomotion strategies have been studied within the framework of optimal control theory and the resulting discrete large-scale Non-Linear Programming problem (NLP) is solved efficiently with the iterative strategy.

2 Computational framework: Optimal control

Lets us consider an undeformed limbless organism with muscles oriented along preferred direction $\boldsymbol{i}$ in the reference configuration (Ω_o). Muscles activity has been modelled as multiplicative decomposition of deformation gradient ($\mathbf{F} := \mathbf{F}_e\mathbf{F}_g$) into inelastic growth ($\mathbf{F}_g := \mathbf{I} + u\boldsymbol{i}\otimes\boldsymbol{i}$ with active growth u) and passive elastic response ($\mathbf{F}_e$ with $J_e := \det(\mathbf{F}_e)$). The worm is modelled as a Neo-Hookean solid with Helmholtz free energy per unit intermediate configuration ψ^e ($:= \frac{\lambda_o}{2}(\ln J_e)^2 + \mu(\mathrm{Tr}(\mathbf{E}_e) - \ln J_e)$, μ and λ_o are shear and bulk modulus, respectively). For thermodynamic consistency, the First Piola-Kirchhoff stress tensor emerged as $\mathbf{P} = J_g \frac{\partial\psi^e}{\partial\mathbf{F}_e}\mathbf{F}_g^{-T}$ (with $J_g := \det(\mathbf{F}_g)$) [2]. Worm-soft substrate interaction is modelled in Ω_o with following anisotropic contact law (μ_t and μ_l friction coefficient along the tangential and lateral direction, respectively)

$$\boldsymbol{t}_o = -\mathbf{B}(\boldsymbol{\tau},\boldsymbol{n},\boldsymbol{v})\boldsymbol{v} \tag{1}$$

$$\mathbf{B}(\boldsymbol{\tau},\boldsymbol{n},\boldsymbol{v}) = \mu_l(\mathbf{I} - \boldsymbol{n}\otimes\boldsymbol{n}) + (\mu_t - \mu_l)(\boldsymbol{\tau}\otimes\boldsymbol{\tau}) \tag{2}$$

$$2\mu_t = \mu_f + \mu_b + \left(\mu_f - \mu_b\right)c(\boldsymbol{v},\boldsymbol{\tau}) \tag{3}$$

where $\mathbf{B}$ is frictional tensor, $\boldsymbol{n}$ is substrate normal, $\boldsymbol{\tau}$ is tangential direction and $\boldsymbol{v}$ is velocity field. $c = \mathrm{sign}(\boldsymbol{v}\cdot\boldsymbol{\tau}) \approx \tanh\left(\frac{\boldsymbol{v}\cdot\boldsymbol{\tau}}{\varepsilon}\right)$, $\varepsilon \approx 0.1$ a Tanh-regularisation parameter. μ_f and μ_b are friction coefficients along the forward and reverse direction, respectively.

At any time 't' in the absence of body and inertia forces, linear momentum balance results in governing Partial Differential Equation (PDE) (ν is a regularisation coefficient). Proceeding to the discretisation of space with finite element (FE), weak form of the PDE is expressed as an Ordinary Differential Equations (ODEs) and written as (where $\tilde{\boldsymbol{x}}$ is nodal position of FE grid)

$$\begin{array}{l} \nu\dot{\boldsymbol{x}} + \nabla_{\boldsymbol{X}}\cdot\mathbf{P} = \mathbf{0} \\ \mathbf{P}\hat{\boldsymbol{N}} = \boldsymbol{t}_o \\ \boldsymbol{x}(\boldsymbol{X},0) = \boldsymbol{x}_0 \end{array} \quad \text{Space discretisation with FE} \quad \begin{array}{l} \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{f}(\tilde{\boldsymbol{x}},u) \\ \tilde{\boldsymbol{x}}(0) = \boldsymbol{x}_0 \end{array} \tag{4}$$

Let us proceed formally towards the optimal control framework to study the optimal locomotion strategies in the limbless organism. Considering this, we aim to solve the following minimisation problem

$$\min_{\tilde{\boldsymbol{x}},\boldsymbol{u}} J(\tilde{\boldsymbol{x}},\boldsymbol{u}) \tag{5}$$

$$\begin{array}{lr} \text{s.t.,} & \\ \quad \boldsymbol{f}(\tilde{\boldsymbol{x}},\boldsymbol{u}) - \dot{\tilde{\boldsymbol{x}}} = \mathbf{0} & \text{(State ODE)} \\ \quad \tilde{\boldsymbol{x}}(0) - \boldsymbol{x}_0 = \mathbf{0} & \text{(Initial conditions)} \\ \quad \boldsymbol{u} \in [\mathscr{U}_{\min}, \mathscr{U}_{\max}] & \text{(Control bounds)} \end{array} \tag{6}$$

Below performance index has been used in this study ($\boldsymbol{x}_{cm}$ represents the centre of mass position at time t, $\boldsymbol{x}_d$ is the desired centre of mass position, T is the final time, and $\alpha \approx 10^{-5}$ penalises the magnitude of muscle growth ($\boldsymbol{u}$)).

$$J(\tilde{\boldsymbol{x}},\boldsymbol{u}) := \int_0^T \left(\frac{1}{2}(\boldsymbol{x}_{cm} - \boldsymbol{x}_d)^\mathsf{T}(\boldsymbol{x}_{cm} - \boldsymbol{x}_d) + \frac{\alpha}{2}\boldsymbol{u}^\mathsf{T}\boldsymbol{u}\right)dt + \frac{1}{2}(\boldsymbol{x}_{cm}(T) - \boldsymbol{x}_d)^\mathsf{T}(\boldsymbol{x}_{cm}(T) - \boldsymbol{x}_d) = \int_0^T (r(\tilde{\boldsymbol{x}}) + q(\boldsymbol{u}))\,dt + \phi(\tilde{\boldsymbol{x}}(T)) \tag{7}$$

We introduce Lagrangian multipliers ($\boldsymbol{\lambda}$, and $\boldsymbol{\xi}$) and define an augmented Lagrangian functional (assuming $\boldsymbol{u} \in [\mathscr{U}_{\min}, \mathscr{U}_{\max}]$)

$$\mathscr{L}(\boldsymbol{x}, \boldsymbol{u}; \boldsymbol{\lambda}, \boldsymbol{\xi}) := \int_0^T \left(\mathscr{H}(\boldsymbol{x}, \boldsymbol{u}; \boldsymbol{\lambda}) - \boldsymbol{\lambda}^{\mathsf{T}} \dot{\tilde{\boldsymbol{x}}} \right) dt + \boldsymbol{\xi}^{\mathsf{T}} (\tilde{\boldsymbol{x}}(0) - \boldsymbol{x}_0) + \phi(\tilde{\boldsymbol{x}}(T)) \tag{8}$$

where $\mathscr{H}(\tilde{\boldsymbol{x}}, \boldsymbol{u}; \boldsymbol{\lambda}) := r(\tilde{\boldsymbol{x}}) + q(\boldsymbol{u}) + \boldsymbol{f}(\tilde{\boldsymbol{x}}, \boldsymbol{u})^{\mathsf{T}} \boldsymbol{\lambda}$ is known as control Hamiltonian.

Optimality conditions results in adjoint equations ($\dot{\boldsymbol{\lambda}} = -\nabla_{\tilde{\boldsymbol{x}}} \mathscr{H}$), ordinary differential equations ($\dot{\tilde{\boldsymbol{x}}} = \nabla_{\boldsymbol{\lambda}} \mathscr{H}$), and control equation ($\boldsymbol{u} = \min(\mathscr{U}_{\max}, \max(\mathscr{U}_{\min}, \underset{\tilde{\boldsymbol{u}}}{\operatorname{argmin}} \mathscr{H}(\tilde{\boldsymbol{x}}(\tilde{\boldsymbol{u}}), \boldsymbol{\lambda}(\tilde{\boldsymbol{u}}), \tilde{\boldsymbol{u}})))$ with two-point boundary conditions ($\tilde{\boldsymbol{x}}(0) = \boldsymbol{x}_0$ and $\boldsymbol{\lambda}(T) = \nabla_{\tilde{\boldsymbol{x}}} \phi(\tilde{\boldsymbol{x}}(T))$). Optimality conditions have been efficiently solved with Forward-Backward Sweep Method (FBSM) [3,4].

3 Numerical Examples

We present two numerical examples to examine the universality of the proposed computational framework. The first example consists of the generation of distinct gait patterns performed by caterpillar (inching), maggot (crawling) and C. elegans (undulatory). For the first two gaits, frictional anisotropy is along the forward and reverse direction of advancement, however, for C. elegans it is along the direction of advancement and direction normal to it (see Fig. 1a). The next example seeks optimal control (growth distribution) which propels C. elegan centre of mass from reference configuration ($\boldsymbol{x}_{cm} = \{5, 0, 0\}$) to the desired target location ($\boldsymbol{x}_d = \{1, 0, 0\}$) in 4s. FBSM is initiated by assigning growth $\boldsymbol{u} = 0.01 \in [-0.3, 0.3]$ to active muscles (FE cells) and optimal control predicts a travelling growth wave opposite to the direction of advancement (see Fig. 1b).

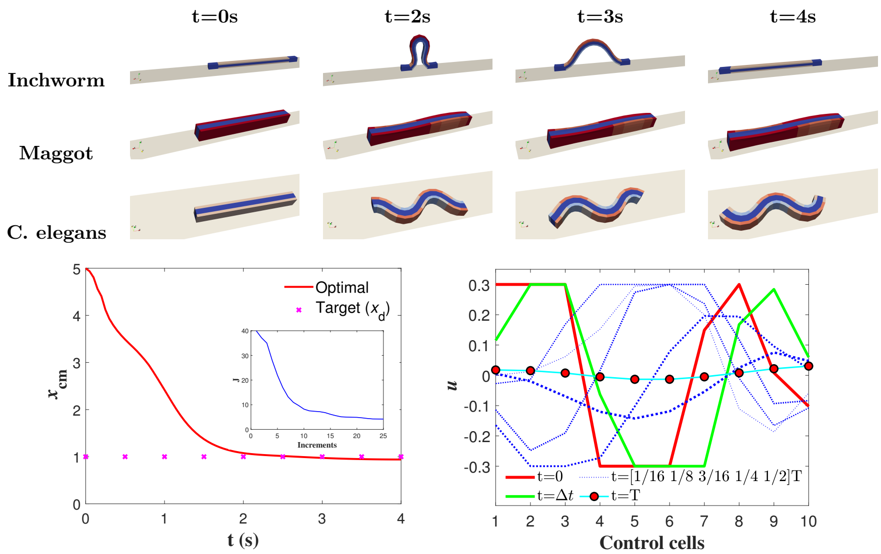

Figure 1: Limbless locomotion on soft substrate: (a) distinct gait patterns (with forward dynamics), and (b) optimal locomotion of C. elegans: the evolution of centre of mass ($\boldsymbol{x}_{cm}$) and growth distribution ($\boldsymbol{u}$).

4 Conclusion

In this work, we have presented a novel computational framework for locomotion in the limbless organism. Central ideas have been validated and applied to the challenging problems of soft robotics. In future, we will extend the proposed framework to study the optimal locomotion pattern in inchworms and maggot-like gait.

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References

[1] Z. Juhasz, A. Zelei. Analysis of worm-like locomotion. Periodica Polytechnica Mechanical Engineering, 57(2), 2013.

[2] J. Bonet, R. D. Wood. Nonlinear continuum mechanics for finite element analysis. Cambridge university press, 1997.

[3] S. Lenhart, J.T. Workman. Optimal control applied to biological models. Chapman and Hall, 2007.

[4] A. Bijalwan, J.J. Muñoz. A control Hamiltonian preserving discretisation for optimal control. Multibody System Dynamics, 2023. In press.