

# NMPC-based Control of Overdetermined Systems by the Example of Magnet Control of the Transrapid

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## EXTENDED ABSTRACT

### 1 Introduction

One common challenge in control design is handling overdetermined systems, i.e., systems with more control inputs than degrees of freedom. The example studied throughout this contribution is the so-called bow magnet of the magnetic levitation (Maglev) vehicle Transrapid. The particular challenge can be seen in Figure 1, which illustrates a two-dimensional model of a longitudinal cut through the Transrapid introduced in [4]. The bow levitation magnets placed at the vehicle's front and back represent mechatronic systems with two mechanical degrees of freedom, while, due to their increased length with respect to the standard levitation magnets, having three actuator inputs. Model predictive control (MPC) as a model-based control strategy is already successfully implemented for standard levitation magnets contributing to the levitation system [2]. Therefore, this work studies the application of nonlinear MPC methods at the bow magnet being an overdetermined system. A classical implementation using one individual control algorithm per control input of the magnet is compared to a newly developed control scheme of one centralized controller for the whole magnet. Advantages, as well as disadvantages of both schemes are discussed.

### 2 Magnet Modeling

Model-based control techniques are based on simulation models, covering the underlying system dynamics to a level of detail suitable for the design method of choice. In the case of the Transrapid's levitation magnets, surrogate models have to be found since the vehicle model as displayed in Figure 1 cannot directly be used for control design. For individual control of each magnet part one usually approximates the system behavior by a magnetically levitated point mass as demonstrated in [2]. This approach will be taken over to the bow magnet by dividing it into three levitated point masses, which however are no longer independently movable due to the rigidity of the magnet body. For designing a centralized controller taking these dependencies into account, a more detailed model as described below is set up which is illustrated in Figure 2. The modeling of the magnet's mechanics with two degrees of freedom  $z$  and  $\varphi$  follows the rigid multibody system approach and is realized with aid of the Matlab-based toolbox Neweul-M<sup>2</sup> [1]. One concentrated magnet force  $F_i, i \in \{a, b, c\}$  per magnet part lifts the magnet body, while additional loads  $F_{AL,j}, j \in \{1, 2, 3\}$  at the magnet's support points apply the expected static load of the vehicle. Furthermore, reference points on the guideway, interacting with the magnet forces and measurement units, are movable in  $z$ -direction to apply external disturbances to the system as they are expected, e.g., from bending and unevenness of the guideway. Finally, both models, the levitated point mass model and the mechanical model of the bow magnet are coupled with the actual magnets' electrodynamics which provides the magnet force dependent on its internal state and input voltage  $U_i$  provided by the controller. The bow magnets' electromagnetic model follows the approach introduced in [3], from which simplified models in form of ordinary differential equations (ODEs) are derived. Although simplified, these ODE models still cover all necessary physical effects such as magnetic saturation, leakage fluxes, and eddy currents. For simulation and analysis the magnet's mechanics, its electrodynamics, and the designed control units are coupled within Simulink as illustrated in Figure 2.

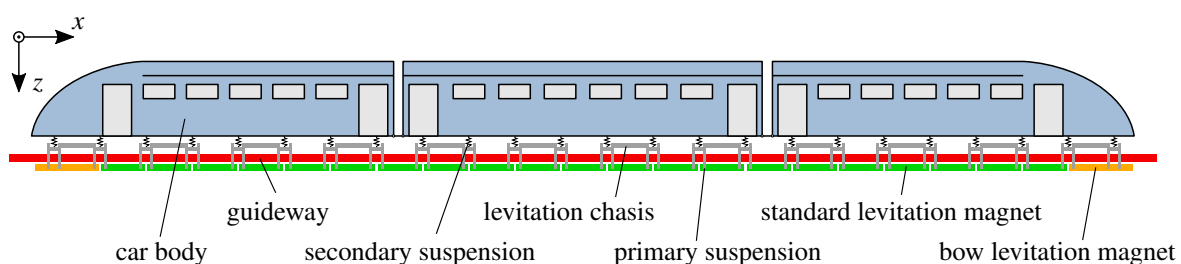


Figure 1: Large-scale vehicle model providing a highly realistic simulation environment for the investigations regarding the control of the bow magnets installed at the vehicle's front and rear (marked yellow). Figure adapted from [4].

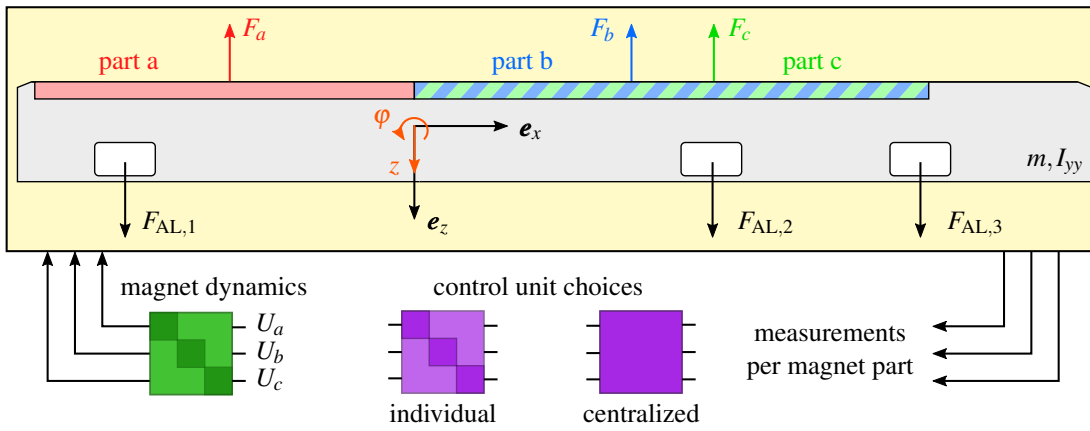


Figure 2: Modeling of the Transrapid's bow levitation magnet as rigid body and integration into the simulation model by coupling of all mechatronic components in Simulink.

### 3 Control Design

The design of model predictive controllers takes advantage over classical control schemes by taking the model's nonlinearities into account and at the same time naturally is handling constraints, e.g., given by actuator limitations. Therefore, the previously derived models of the magnet's mechanics and electrodynamics are coupled manually and incorporated into the design procedure. For the individual control units, the levitated point mass model is coupled with one ODE for the magnetic behavior of said magnet part leading to a prediction model with three states and one input constraint. In contrast, the centralized controller's prediction model consists of the full rigid body model of the magnet and three ODEs for the magnetic behavior resulting in a model with seven states and three input constraints. Both control schemes investigated in this contribution assume measurement of all state variables required by the respective algorithm. Observer techniques or state filtering may be applied to counter this idealizing assumption and will be necessary for implementation in the real system. Furthermore, measures against steady-state tracking error have to be taken and are realized by an integral error approach as proposed in [2], again increasing the dimension of the internal models by integral states.

### 4 Results

First results are obtained with the model of the levitated bow magnet in its isolated environment as displayed in Figure 2. While the performance is indistinguishable in nominal operation, the centralized control approach exploits its information about the whole system state in certain scenarios, such as during static measurement uncertainty or failure of a magnet part. In such scenarios, independent control units lack performance and under certain conditions even act contrary. Further investigations regarding stability and ride comfort are carried out within the large-scale vehicle model as shown in Figure 1. The vehicle model was presented in [4] and represents a longitudinal cut through the vehicle, mapping the heave and pitch motions of every involved component while riding on an elastic guideway of infinite length, thus providing a highly realistic simulation environment. Again, the centralized control strategy is shown to be beneficial in scenarios beyond the nominal operation. While the newly developed control scheme leads to promising results in simulation, implementation in the real-world system requires two further aspects to be taken into account. For once, the real-time capability of the MPC problem with increased size has to be proven with the desired control hardware and finally, the lack of redundancy with the centralized controller in comparison to independently acting control units has to be compensated.

### References

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