

# Some studies on combined feedforward and funnel control of underactuated multibody systems

Robert Seifried<sup>1</sup>, Svenja Drücker<sup>1</sup>, Thomas Berger<sup>2</sup>, Lukas Lanza<sup>3</sup>, Timo Reis<sup>3</sup>

<sup>1</sup> Institute of Mechanics and Ocean Engineering  
Hamburg University of Technology  
{robert.seifried,svenja.druecker}@tuhh.de

<sup>2</sup> Institute for Mathematics  
Paderborn University  
thomas.berger@upb.de

<sup>3</sup>Institute for Mathematics  
Ilmenau University of Technology  
{lukas.lanza,timo.reis}@tu-ilmenau.de

## EXTENDED ABSTRACT

### 1 Introduction

Underactuation occurs naturally in the design of many mechanical systems, such as light-weight machines, many types of cable-driven manipulators or flexible joint robots. Such systems have more degrees of freedom than independent control inputs. With advances in the mechanical design, there is also a need for new control strategies. However, it is usually not possible to control all degrees of freedom independently due to lack of actuators. Therefore, control of underactuated systems is a challenging problem [1].

This contribution analyzes a two degree of freedom control strategy for underactuated systems. The control strategy combines funnel control with feedforward control based on servo-constraints. Simulation and experimental results are presented in order to show the effects of the control methodology.

### 2 Control Methodology

For trajectory tracking, a two degree of freedom control structure is an efficient method for underactuated systems. A feedforward controller moves it along a prescribed trajectory and a feedback controller rejects disturbances and reduces remaining tracking errors.

In the approach taken here, the method of servo-constraints is applied to compute an inverse model of the underactuated system [2]. The inverse model can be directly used in the feedforward control loop. This model-based control strategy has proven to be an efficient method for complex underactuated systems [3].

In the feedback loop, funnel control is utilized, which is an adaptive output feedback control strategy originally designed in [5]. It is a model-free and therefore robust controller which is able to guarantee prescribed performance of the tracking error even in the presence of uncertainties and disturbances. The tracking error stays within a defined performance funnel.

In the control approach taken here, both parts of the controller can first be designed and tested independently. Afterwards, they are combined to retain the advantages of both methods.

### 3 Results

The methodology is validated for two application examples. Firstly, simulation results are presented for a mass-on-car system. Secondly, experimental results are shown for a torsional oscillator.

**Mass-on-car System.** As first application example, the mass-on-car system shown in Fig. 1a is considered. It consists of two masses connected by a linear spring-damper combination. The system input  $u$  is a force applied on the first mass and the output is the horizontal position of the second mass. The system is therefore underactuated with one unactuated degree of freedom. In order to show the influence of the feedback control, the inverse model is computed for a nominal model. The feedforward control input is then applied to a model with disturbed parameters. The funnel feedback controller is applied in order to reduce the tracking errors due to the disturbed parameters. The simulation results in Fig. 1b show that the funnel control itself, denoted by  $\mathbf{u}_{fb}$  experiences peaks in the control signal. These peaks can be reduced with the combination of funnel control and servo-constraints, denoted by  $\mathbf{u}_{fb} + \mathbf{u}_{ffw}$ . In this case, a significant part of the control signal is due to the inverse model and noise effects are reduced in the feedback loop [4].

**Torsional Oscillator.** As a second application example, the torsional oscillator shown in Fig. 2a is considered. It is an experimental setup with two rotating disks which are connected by a thin rod. The lower mass is actuated and is also considered as system output in this example. Therefore, the second mass can be interpreted as a disturbance on the output. The desired trajectory is a smooth transition from 0 rad/s to 6.28 rad/s (2 revolutions per second). Figure 2b shows the experimental results with the funnel control by itself. There is a time lag and a steady state error of the signal with respect to the reference. The combination with an inverse model can reduce both these effects, see Fig. 2c.

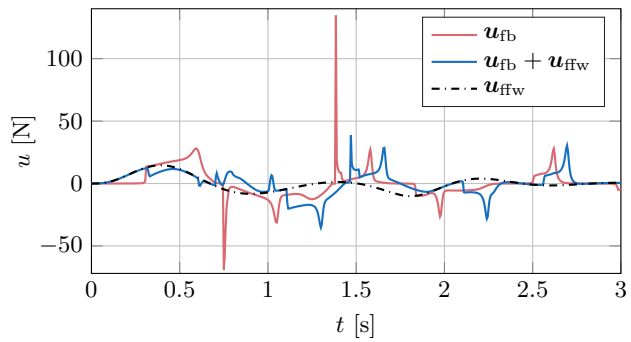
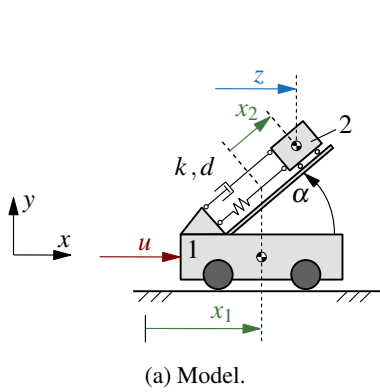


Figure 1: Simulation results for the mass-on-car system.

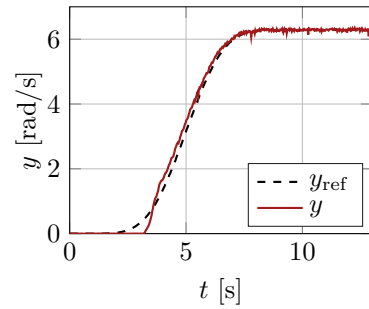
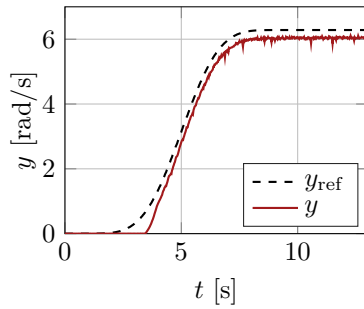
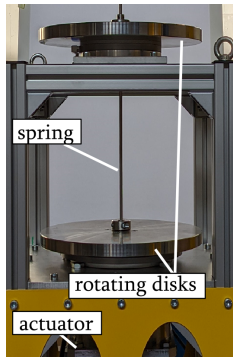


Figure 2: Experimental results for the torsional oscillator.

#### 4 Conclusion

The simulation as well as experimental results show that the combination of servo-constraints with funnel control retains the advantages of both individual methods. First of all, simulation results demonstrate that the feedforward control part reduces peaks in the funnel feedback control signal. This reduces loads on the mechanical parts of the system. Moreover, experimental results with a torsional oscillator show that the feedforward control part is necessary to obtain a small steady state error after reaching the final position.

#### Acknowledgments

This work was supported by the German Research Foundation (Deutsche Forschungsgemeinschaft) via the grant number 396289190.

#### References

- [1] R. Seifried, Dynamics of Underactuated Multibody Systems Modeling, Control and Optimal Design. Cham s.l.: Springer International Publishing, 2014.
- [2] W. Blajer and K. Kołodziejczyk, “A Geometric Approach to Solving Problems of Control Constraints: Theory and a DAE Framework,” *Multibody System Dynamics*, vol. 11, Art. no. 4, 2004.
- [3] S. Otto and R. Seifried, “Real-time trajectory control of an overhead crane using servo-constraints,” *Multibody System Dynamics*, vol. 42, Art. no. 1, Jan. 2018.
- [4] T. Berger, S. Otto, T. Reis, and R. Seifried, “Combined open-loop and funnel control for underactuated multibody systems,” *Nonlinear Dynamics*, vol. 95, pp. 1977–1998, Dec. 2018, doi: 10.1007/s11071-018-4672-5.
- [5] A. Ilchmann, E. P. Ryan, and C. J. Sangwin, “Tracking with prescribed transient behaviour,” *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 7, pp. 471–493, 2002.