Modeling the ankle articular complex: a computational and experimental analysis

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EXTENDED ABSTRACT

1 Introduction

The foot is a complex system that allows several movements of the daily life. Amongst its 31 anatomical articulations, the talocrural and the talocalcaneal ones, which collectively form the ankle articular complex of the human foot, are the focal point of this work. The axes of these articulations are non-coplanar, and the distance is defined as the height of the talus [1]. Literature review indicates that the ankle articular complex can be modeled in distinct ways [2,3]. Most approaches do not consider a sufficiently accurate anatomical modeling of this articular complex. Hence, the aim of this work is to present a new formulation of the ankle articular complex of the human foot and provide a comparison between computational and experimental approaches.

2 Formulation to model the ankle articular complex

In this work, a modified universal joint (Figure 1a) is proposed for the ankle articular complex of the human foot. The proposed approach, incorporated with a massless link representing the talus, allows two relative degrees-of-freedom (DoF) between the connected bodies. Thus, four kinematic constraint equations must be considered to formulate this joint as

$$\boldsymbol{\Phi}^{(\mathrm{u_m},4)} = \begin{cases} \mathbf{d}^{\mathrm{T}}\mathbf{d} - l^2 = 0 \\ \mathbf{s}_i^{\mathrm{T}}\mathbf{s}_j - \mathbf{s}_{i_0}^{\mathrm{T}}\mathbf{s}_{j_0} = 0 \\ \mathbf{s}_i^{\mathrm{T}}\mathbf{d} - \mathbf{s}_{i_0}^{\mathrm{T}}\mathbf{d}_0 = 0 \\ \mathbf{s}_j^{\mathrm{T}}\mathbf{d} - \mathbf{s}_{j_0}^{\mathrm{T}}\mathbf{d}_0 = 0 \end{cases} \tag{1}$$

where $\mathbf{s}_i$ and $\mathbf{s}_j$ represent the vectors of the joint's axes, l denotes the length of the massless link, and $\mathbf{d}$ is established as

$$\mathbf{d} = \mathbf{r}_j^P - \mathbf{r}_i^P = \mathbf{r}_j + \mathbf{s}_j^P - \mathbf{r}_i - \mathbf{s}_i^P \tag{2}$$

in which $\mathbf{r}_k^P$ is the global position vector of point P located on body k (k=i, j), $\mathbf{r}_k$ denotes the position vector of the center of mass of body k described in global coordinates and $\mathbf{s}_k^P$ is the global position vector of point P located on body k with respect to the body's local coordinate system [4]. Vectors $\mathbf{s}_{i_0}$, $\mathbf{s}_{j_0}$ and $\mathbf{d}_0$ represent the initial coordinates of vectors $\mathbf{s}_i$, $\mathbf{s}_j$ and $\mathbf{d}$, respectively. The contribution to the Jacobian matrix of the modified universal joint constraints can be represented as

$$\mathbf{D}^{(\mathrm{u_m},4)} = \begin{bmatrix} -2\mathbf{d}^{\mathrm{T}} & 2\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{s}}_i^P & 2\mathbf{d}^{\mathrm{T}} & -2\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{s}}_j^P \\ \mathbf{0} & -\mathbf{s}_j^{\mathrm{T}}\tilde{\mathbf{s}}_i & \mathbf{0} & -\mathbf{s}_i^{\mathrm{T}}\tilde{\mathbf{s}}_j \\ -\mathbf{s}_i^{\mathrm{T}} & -\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{s}}_i + \mathbf{s}_i^{\mathrm{T}}\tilde{\mathbf{s}}_i^P & \mathbf{s}_i^{\mathrm{T}} & -\mathbf{s}_i^{\mathrm{T}}\tilde{\mathbf{s}}_j^P \\ -\mathbf{s}_j^{\mathrm{T}} & \mathbf{s}_j^{\mathrm{T}}\tilde{\mathbf{s}}_i^P & \mathbf{s}_j^{\mathrm{T}} & -\mathbf{d}^{\mathrm{T}}\tilde{\mathbf{s}}_j - \mathbf{s}_j^{\mathrm{T}}\tilde{\mathbf{s}}_j^P \end{bmatrix} \tag{3}$$

Finally, the right-hand side vector of the acceleration constraint equations of the modified universal joint can be established as

$$\boldsymbol{\gamma}^{(\mathrm{u_m},4)} = \begin{Bmatrix} 2\mathbf{d}^{\mathrm{T}}(\tilde{\boldsymbol{\omega}}_i\dot{\mathbf{s}}_i^P - \tilde{\boldsymbol{\omega}}_j\dot{\mathbf{s}}_j^P) - 2\dot{\mathbf{d}}^{\mathrm{T}}\dot{\mathbf{d}} \\ -\mathbf{s}_i^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_j\dot{\mathbf{s}}_j - \mathbf{s}_j^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_i\dot{\mathbf{s}}_i - 2\dot{\mathbf{s}}_j^{\mathrm{T}}\dot{\mathbf{s}}_i \\ -\mathbf{d}^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_i\dot{\mathbf{s}}_i - 2\dot{\mathbf{d}}^{\mathrm{T}}\dot{\mathbf{s}}_i - \mathbf{s}_i^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_j\dot{\mathbf{s}}_j^P + \mathbf{s}_i^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_i\dot{\mathbf{s}}_i^P \\ -\mathbf{d}^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_j\dot{\mathbf{s}}_j - 2\dot{\mathbf{d}}^{\mathrm{T}}\dot{\mathbf{s}}_j - \mathbf{s}_j^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_j\dot{\mathbf{s}}_j^P - \mathbf{s}_j^{\mathrm{T}}\tilde{\boldsymbol{\omega}}_i\dot{\mathbf{s}}_i^P \end{Bmatrix} \tag{4}$$

in which the dot represents the derivative with respect to time, $\boldsymbol{\omega}$ is the angular velocity vector in global coordinates, and the symbol (~) denotes the skew symmetric matrix.

3 Guiding constraints formulation

With the purpose of prescribing the acquired experimental data to the multibody models, two types of driving constraints are formulated, namely guiding (*i*) a generic point G and (*ii*) an arbitrary Euler parameter to control the orientation of a body.

To guide a generic point G, the following three kinematic constraints are considered

$$\mathbf{\Phi}^{(G,3)} = \mathbf{r}_i^G - \mathbf{c}^*(t) = \mathbf{r}_i + \mathbf{s}_i^G - \mathbf{c}^*(t) = \mathbf{0} \tag{5}$$

where $\mathbf{c}^*(t)$ is the prescribed experimental data containing the x, y, and z coordinates of the guided point and t is the time variable. The corresponding contribution to the Jacobian matrix, the right-hand side vectors of the velocities and acceleration are as

$$\mathbf{D}^{(G,3)} = \begin{bmatrix} \mathbf{I} & -\tilde{\mathbf{s}}_i^G \end{bmatrix} \tag{6}$$

$$\mathbf{v}^{(G,3)} = -\frac{\partial \mathbf{\Phi}^{(G,3)}}{\partial t} = \dot{\mathbf{c}}^*(t) \tag{7}$$

$$\boldsymbol{\gamma}^{(G,3)} = \tilde{\boldsymbol{\omega}}_i \tilde{\mathbf{s}}_i^G \boldsymbol{\omega}_i + \ddot{\mathbf{c}}^*(t) \tag{8}$$

in which $\mathbf{I}$ represents the identity matrix.

To guide one arbitrary Euler parameter, e_k, with k = 0, 1, 2, 3, the following constraint equation, the corresponding Jacobian matrix, and the right-hand side vectors of the velocities and acceleration are utilized

$$\Phi^{(e_k,1)} = e_k - e_k^*(t) = 0 \tag{9}$$

$$\mathbf{D}^{(e_k,1)} = \begin{bmatrix} \mathbf{0} & \frac{1}{2}\left(\mathbf{G}_{(1:3,k+1)}\right)^{\mathrm{T}} \end{bmatrix} \tag{10}$$

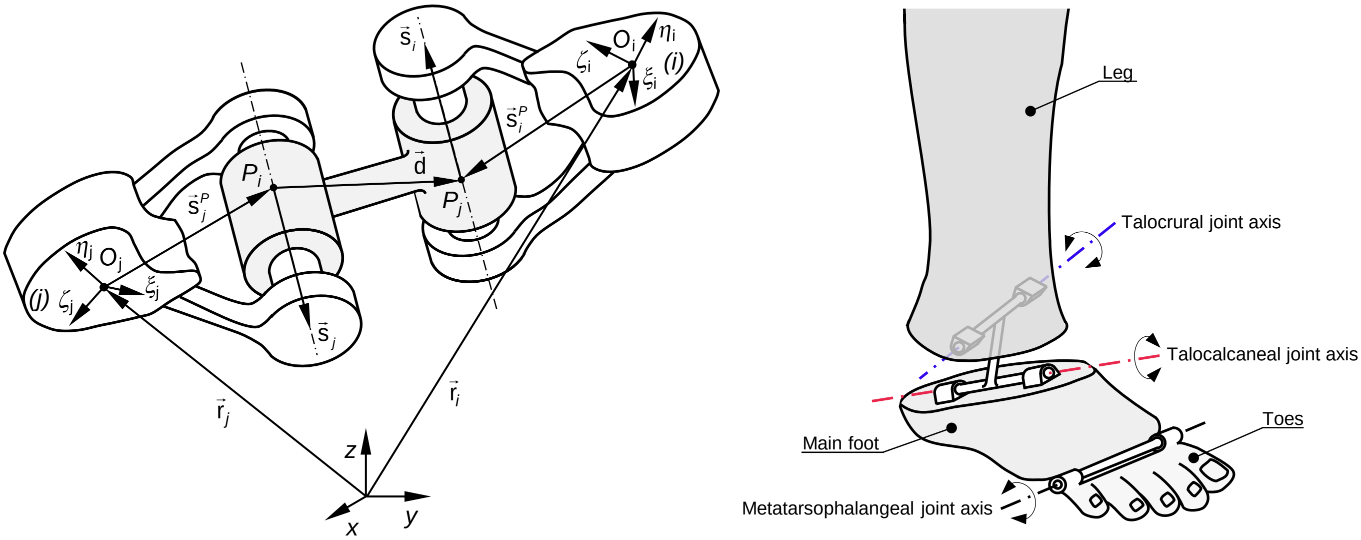
$$\nu^{(e_k,1)} = -\frac{\partial \mathbf{\Phi}^{(e_k,1)}}{\partial t} = \dot{e}_k^*(t) \tag{11}$$

$$\gamma^{(e_k,1)} = \ddot{e}_k^*(t) - \frac{1}{2}\boldsymbol{\omega}_i^{\mathrm{T}} \dot{\mathbf{G}}_{(1:3,k+1)} \tag{12}$$

where $e_k^*(t)$ is the experimental data for e_k and $\mathbf{G}$ represents the transformation matrix in terms of the Euler parameters [4].

4 Results and discussion

A three-dimensional biomechanical model is used in this work (Figure 1b). The model is composed of three rigid bodies, namely the leg, main foot, and toes, which are kinematically connected to each other by one revolute joint, connecting the toes to the main foot (metatarsophalangeal articulations), and one modified universal joint, connecting the main foot to the leg (ankle articular complex). The biomechanical model has nine DoF. Experimental data of one adult subject with no history of gait disorders was acquired at the Lisbon Biomechanics Laboratory and it is used to guide the model's DoF. The obtained results are compared with the literature and the response of the proposed joint model for the ankle articular complex of the human foot is assessed.



(a) (b)

Figure 1 : Schematic representation of (a) the modified universal joint and (b) the considered three-dimensional biomechanical model.

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