Numerical Treatment of Nonsmooth Frictional Beam-to-Beam Contact

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EXTENDED ABSTRACT

1 Introduction

Highly slender structures undergoing large displacements, such as beams, cables or flexible plates are integral parts of most modern high performance engineering systems. Interest in their virtual simulation has strongly increased in recent years [1]. Modeling their contact interactions is especially challenging since on top of the geometric nonlinearities, unilateral contact constraints have to be handled. Most works until now focus on smoothed penalty type approaches [2, 3] with the exception of [4, 5]. In [6], the authors proposed a mortar method for beam-to-beam contact in the static frictionless case. Here, we present an extension to frictional dynamics in the framework of the nonsmooth generalized-α (NSGA) time integration scheme following the developments in [7]. It is combined with the beam finite element formulation on the special Euclidean group [8]. The equations of motion, including the contact forces, are expressed in the local frame attached to the beam centerline and are written on a Lie group resulting in interesting frame invariance properties.

2 Numerical method

Time integration is based on an appropriate Lie group update procedure where the current configuration q_{n+1} is represented by an increment vector $\Delta\mathbf{q}_{n+1}$

$$q_{n+1} = q_n \exp_{SE(3)}(\Delta\mathbf{q}_{n+1}). \tag{1}$$

A splitting of position and velocity into smooth and nonsmooth contributions is performed such that the solution to the following sequence of subproblems needs to be found.

- Smooth prediction of $\widetilde{q}_{n+1}, \widetilde{\mathbf{v}}_{n+1}, \dot{\widetilde{\mathbf{v}}}_{n+1}$:

$$\mathbf{M}(\widetilde{q}_{n+1})\dot{\widetilde{\mathbf{v}}}_{n+1} - \mathbf{f}(\widetilde{q}_{n+1}, \widetilde{\mathbf{v}}_{n+1}, t_{n+1}) = \mathbf{0}. \tag{2}$$

- Correction of the position increment $\Delta\mathbf{q}_{n+1} = \Delta\widetilde{\mathbf{q}}_{n+1} + \mathbf{U}_{n+1}$ by enforcing the frictional contact constraints at postion level via the Lagrange multiplier $\boldsymbol{\nu}_{n+1}$:

$$\mathbf{M}(\widetilde{q}_{n+1})\mathbf{U}_{n+1} - h^2\mathbf{f}^p_{n+1} - \mathbf{g}^T_{q,n+1}\boldsymbol{\nu}_{n+1} = \mathbf{0}, \tag{3a}$$

$$-g^j_{N,n+1} \in \partial\psi_{\mathbb{R}^+}(\nu^j_{N,n+1}), \tag{3b}$$

$$-\mathbf{g}^j_{T,n+1} \in \partial\psi_{C(\nu^j_{N,n+1})}(\boldsymbol{\nu}^j_{T,n+1}), \quad \text{if } g^j_{N,n+1} \leq 0. \tag{3c}$$

- Computation of the velocity jump $\mathbf{v}_{n+1} = \widetilde{\mathbf{v}}_{n+1} + \mathbf{W}_{n+1}$ by solving a contact problem formulated at velocity level with the multiplier $\boldsymbol{\Lambda}_{n+1}$:

$$\mathbf{M}(q_{n+1})\mathbf{W}_{n+1} - h\mathbf{f}^*_{n+1} - \mathbf{g}^T_{q,n+1}\boldsymbol{\Lambda}_{n+1} = \mathbf{0}, \tag{4a}$$

$$-\mathbf{g}^j_{Nq,n+1}\mathbf{v}^j_{n+1} \in \partial\psi_{\mathbb{R}^+}(\Lambda^j_{N,n+1}), \quad \text{if } g^j_{N,n+1} \leq 0, \tag{4b}$$

$$-\mathbf{g}^j_{Tq,n+1}\mathbf{v}^j_{n+1} \in \partial\psi_{C(\Lambda^j_{N,n+1})}(\boldsymbol{\Lambda}^j_{T,n+1}), \quad \text{if } g^j_{N,n+1} \leq 0. \tag{4c}$$

For Eq. (2) the usual generalized-α integration formulae are employed. For Eqs. (3) and (4) we use an augmented Lagrangian technique as in [9] combined with a semismooth Newton algorithm. The discretized mortar constraints are written in the local contact frame $\{\mathbf{n}_n, \mathbf{t}_{1,n}, \mathbf{t}_{2,n}\}$ which is kept fixed for the duration of the time step

$$\mathbf{g}^j_{n+1} = \begin{bmatrix} g^j_{N,n+1} \\ \mathbf{g}^j_{T,n+1} \end{bmatrix} = \int_{\gamma_c} \boldsymbol{\phi}^j_{n+1} \begin{bmatrix} \mathbf{n}^T_n(\mathbf{x}_{OF,n+1} - \mathbf{x}_{OC,n+1}) \\ \mathbf{t}^T_{1,n}(\Delta\mathbf{x}_{OF,n+1} - \Delta\mathbf{x}_{OC,n+1}) \\ \mathbf{t}^T_{2,n}(\Delta\mathbf{x}_{OF,n+1} - \Delta\mathbf{x}_{OC,n+1}) \end{bmatrix} \mathrm{d}s \tag{5}$$

and the corresponding constraint gradient is

$$\mathbf{g}^j_{q,n+1} = \begin{bmatrix} \mathbf{g}^j_{Nq,n+1} \\ \mathbf{g}^j_{Tq,n+1} \end{bmatrix} = \int_{\gamma_c} \boldsymbol{\phi}^j_{n+1} \mathbf{S}_{L,n+1} \mathbf{Q}^j_{n+1} \, \mathrm{d}s, \tag{6}$$

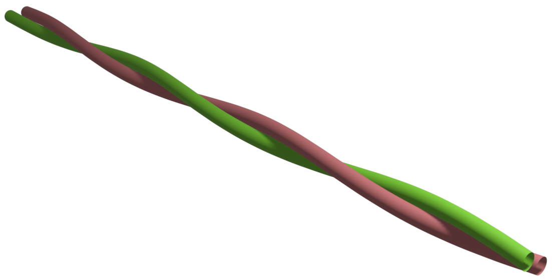
where the operator $\mathbf{S}_L$ takes quantities from the local frames associated to the beam centerline to express them in the local contact frame attached to the surface of the beam

$$\mathbf{S}_{L,n+1} = \mathbf{R}^T_{OL,n} \begin{bmatrix} -\mathbf{R}_{OC,n+1} & \mathbf{R}_{OF,n+1} \end{bmatrix} \mathbf{M} \tag{7}$$

and where $\mathbf{M}$ depends on the shape of the cross-section.

3 Preliminary results

A first comparison of collocation type and mortar contact formulations, with and without friction is illustrated in Fig. 1. The proposed methodology is able to deal with discontinuous velocities stemming from impacts and stick-slip transitions. All beam-to-beam contact algorithms were implemented in the research code Odin [10].



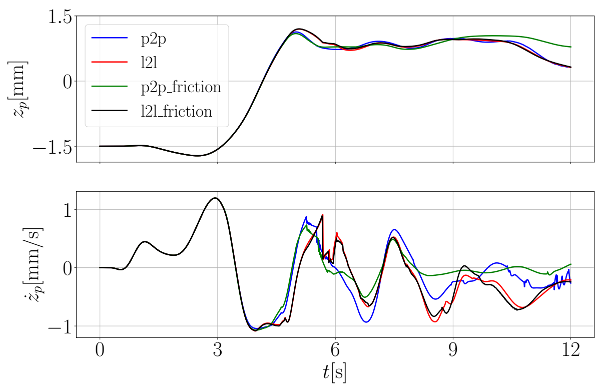


Figure 1: Dynamic twisting example: Time evolution of the vertical component of position and velocity of the beam middle point.

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