

# Analysis of Systems with Viscoelastic Materials Based on Generalized Alpha Scheme

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## EXTENDED ABSTRACT

### 1 Introduction

Rubber bushes are used in automobile suspension systems in order to absorb vibration. They are made from a viscoelastic polymer, meaning that they exhibit both viscous and elastic properties. When a vehicle is running, the wheel alignment is changed by the deflection of the rubber bushes. The lateral force and camber thrust generated in tires by the change of wheel alignment have a significant effect on vehicle dynamics. Therefore, it is important to consider the characteristics of rubber bushes when analyzing vehicle motion.

Since rubber bushes have very high stiffness, suspensions including rubber bushes are stiff systems [1], meaning that these systems have a mixture of high and low eigenvalues. However, since analysis of these systems is computationally expensive due to higher-order modes, it is desirable to analyze the systems using numerical integration methods which have functions to suppress higher-order modes.

Furthermore, viscoelastic materials have complex mechanical characteristics such as nonlinear mechanical properties and frequency dependence [2]. Such characteristics are often analyzed using the finite element method. However, this method is difficult to apply to driving simulators and other applications that require real-time calculation because it requires an enormous amount of computation. Therefore, in order to consider the viscoelastic properties in real-time analysis, it is necessary to represent the properties accurately and with a simple mechanical model.

Therefore, the aim of this study is to investigate a modeling method suitable for real-time analysis. To this end, the characteristics of a setup with a linkage mechanism containing a viscoelastic test piece were experimentally evaluated. Then, the properties of the setup were analyzed using the generalized alpha method and then compared with the test results to evaluate the calculation performance and accuracy of the analysis method.

### 2 Experimental setup with viscoelastic joint

Details about the experimental setup in this study are shown in Figures 1 and 2. Figures 1 (a) and (b) show the general experimental setup and its schematic representation. The setup has an open-loop linkage mechanism with a viscoelastic joint attached between the links. Link 1 is rotated by a DC servomotor. Figure 1 (c) shows the viscoelastic joint in detail, while Figures 2 (a) and (b), respectively show top views of the experimental setup and its schematic representation. The setup focuses on the axial and torsional deformation of a viscoelastic body. The position and rotation of the links were recorded by a motion capture system, while the force and torque on the viscoelastic joint were measured by a 6-axis loadcell attached between the joint and Link 2. Figure 2 (c) shows the time series of the angles of the two links defined in Figure 2 (b). It can be seen that they move in opposite phase.

In addition, the motion capture data was used to calculate the deformation of the viscoelastic joint. Figure 3 shows the axial and torsional deformation of the joint when Link 1 was rotated by the motor with an amplitude of 10 degrees and a frequency of 2 Hz.

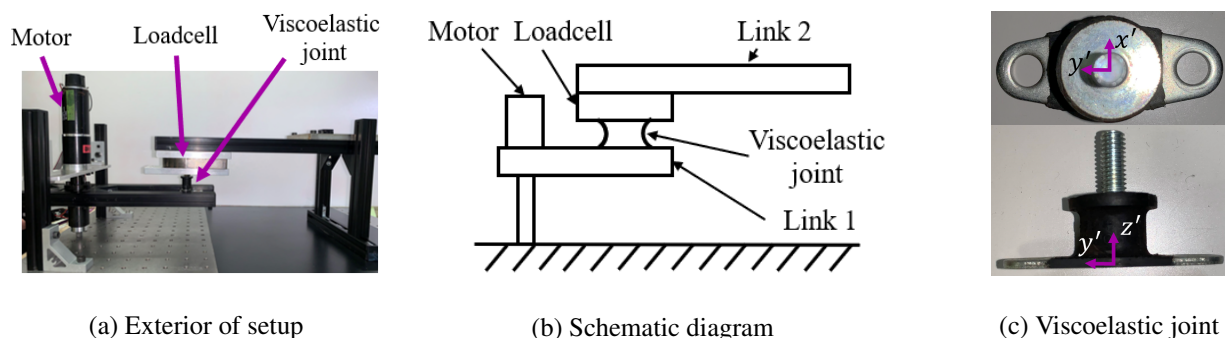
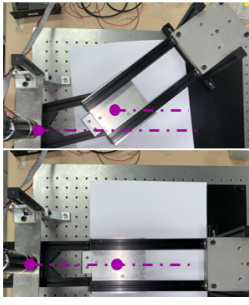
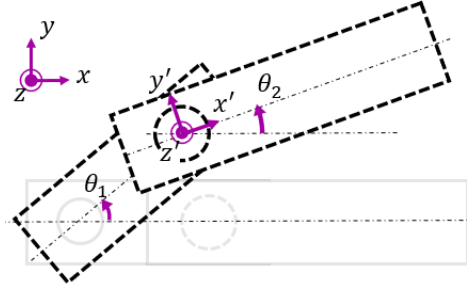


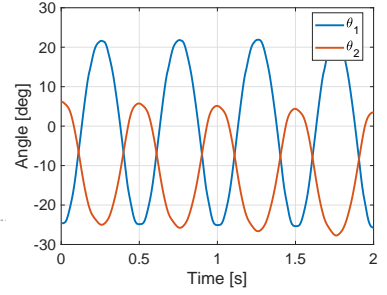
Figure 1: Experimental setup with viscoelastic joint



(a) Exterior of setup (Top view)

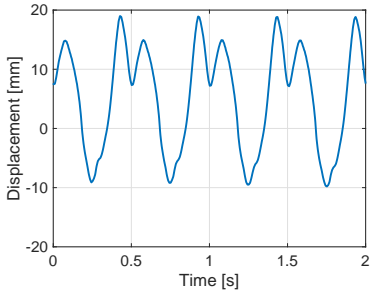


(b) Schematic diagram (Top view)

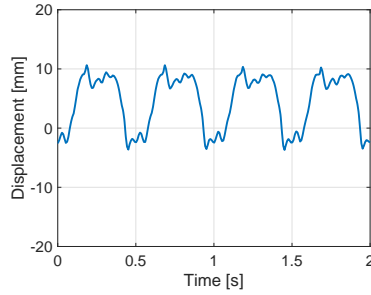


(c) Angle of each link

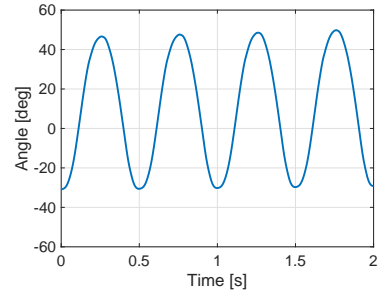
Figure 2: Behavior of the experimental setup



(a)  $x'$ -axis



(b)  $y'$ -axis

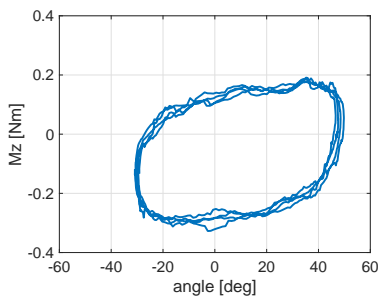


(c) Around  $z'$ -axis

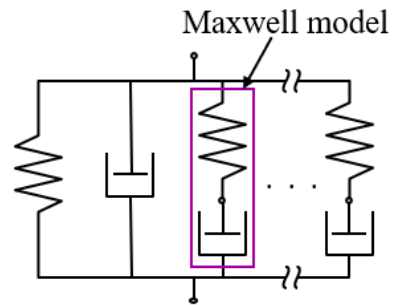
Figure 3: Deformation of the viscoelastic joint

### 3 Hysteresis characteristic of a viscoelastic body

Viscoelastic bodies have both viscous and elastic properties. Several mathematical models with dampers and springs have been proposed to represent the viscosity and the elasticity. Figure 4 (a) shows the relationship between the torsional angle and the torque under the experimental setup described in the previous section. In the current study, we attempted to represent the mechanical properties of the viscoelastic body by the generalized Maxwell model shown in Figure 4 (b). Furthermore, the generalized alpha method was applied for numerical simulation of the experimental setup to suppress unnecessary high-frequency responses by adjusting the parameters  $\alpha_m$  and  $\alpha_f$  which are determined by the spectral radius  $\rho_\infty$  optimally [3].



(a) Torsional moment respect to torsional angle



(b) Generalized Maxwell model

Figure 4: Representation of the mechanical properties of the viscoelastic body

### References

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