

# A Compliant and Redundantly Actuated 3-DOF 4RRR PKM: First Step to Full Planar Motion

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## EXTENDED ABSTRACT

### 1 Introduction

In [1, 2] compliant and redundantly actuated 2-DOF 3RRR parallel kinematic manipulators (PKM) have been introduced and optimised as “best of both worlds” for precision applications. Being compliant mechanisms, or more precisely flexure-based mechanisms, deterministic behaviour can be realised because of the low level of friction, hysteresis and backlash. Being also a redundantly actuated PKM, it combines the advantages of PKM, i.e. higher stiffness, low inertia and large accelerations, with an improved handling of singularities and optimised actuator loading made possible by the redundancy.

Simulations and experimental tests indeed demonstrated advantages of combining both concepts [1]. However a drawback of the developed 2-DOF prototype is the limitation to only translational motion. Hence, in this paper a third degree of freedom is added which is the in-plane rotation of the end effector. The goal is to design a first prototype capable of full planar motion.

### 2 Design Optimisation

The four arms of the 4RRR PKM are assumed to be similar and the actuators are located at the corners of a square. Figure 1(a) shows a simplified kinematic model in which the shoulder, elbow and wrist joints are assumed to be ideal revolute joints. Characteristic dimensions are lengths  $L_1$  and  $L_2$  of upper and lower arm, respectively, as well as length  $L_3$  that represents the dimension of the end effector (EE). The overall dimensions of the system follow from the distance  $R$  between the actuator locations and the centre of the end effector in the presented neutral configuration. Similar to the previous designs  $R = 230.5$  mm [1, 2]. The actuators are direct drive Maxon EC 90 flat brushless motors (part number 323772, nominal torque 0.444 Nm), equipped with a MILE encoder (part number 411966, 6400 pulses/revolution). The motor axle is connected to the upper arm with a flexible coupling (SICK KUP-1010-B). The links and joints are manufactured with 3D printing. For the arms relatively low-cost Fused Deposition Modeling (FDM) of PLA is adequate. The joints are produced with Selective Laser Sintering (SLS) of Nylon (PA2200) that can meet stricter geometric tolerances.

For this first prototype the effective workspace of the manipulator has been limited to avoid rotations larger than  $\pm 30^\circ$  in all flexure joints. From the simplified model in Figure 1(a) it can be understood quite straightforwardly that a rotation of the end effector results in about the same rotation of the wrist joints. To allow combined rotation and translation of the end effector, its rotation is limited to  $\pm 15^\circ$ . Butterfly flexure joints [3], see Figure 1(c), can handle large rotations up to  $\pm 30^\circ$  without a significant loss of support stiffness and are therefore used for the elbow and wrist joints. A cartwheel flexure joint is used for the shoulder as it offers higher, or at least comparable, support stiffness for a smaller range of about  $\pm 15^\circ$ , which suffices for this joint.

To analyse and optimise the manipulator design, two types of models have been used. A simplified dynamic model follows from Figure 1(a) where the ideal revolute joints are assumed to exhibit a constant compliance for the in-plane rotation and infinite stiffness in all other directions. In this way a 3-DOF model is obtained that e.g. can be used to evaluate the three lowest natural frequencies and the required actuator torques. For design optimisation higher natural frequencies associated with unwanted parasitic vibrations must be known as well. Such higher order dynamic behaviour is investigated with a more advanced flexible multibody model in which all flexures are modelled in the SPACAR software package with non-linear beam elements that account for constraint warping [4].

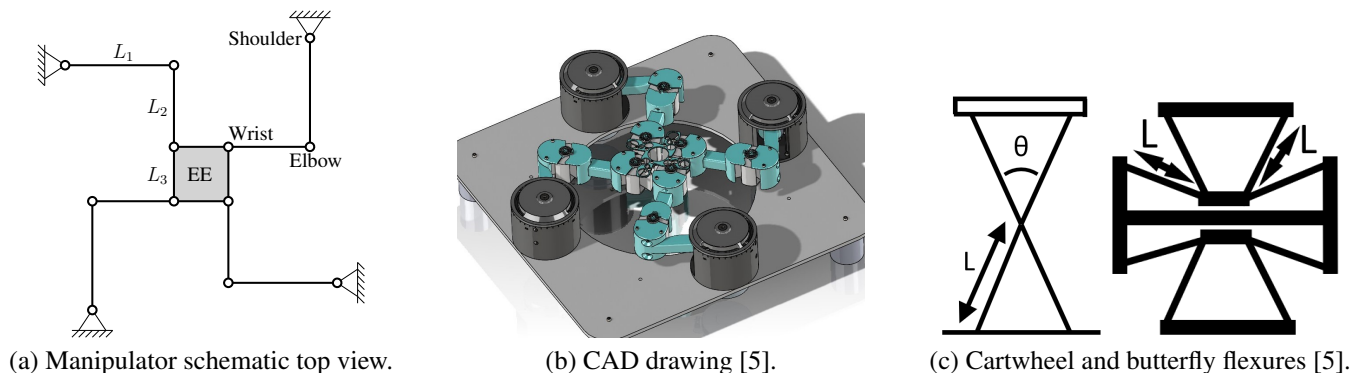


Figure 1: Design of the planar 3-DOF 4RRR PKM with compliant joints.

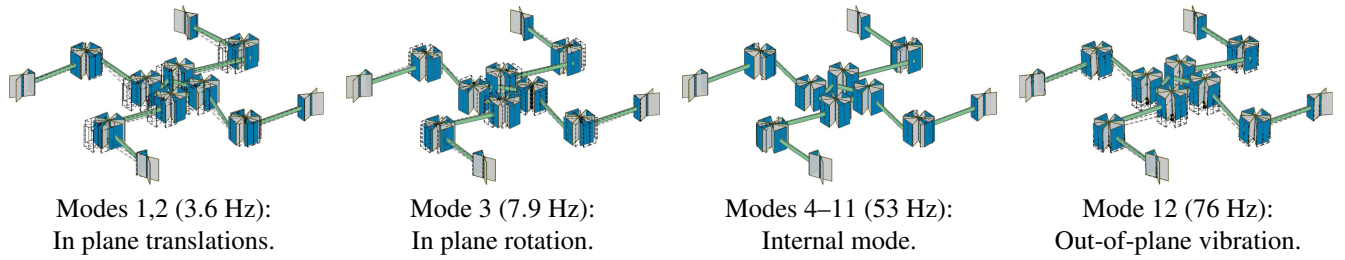


Figure 2: Natural frequencies and mode shapes of the 3-DOF 4RRR PKM with compliant joints.

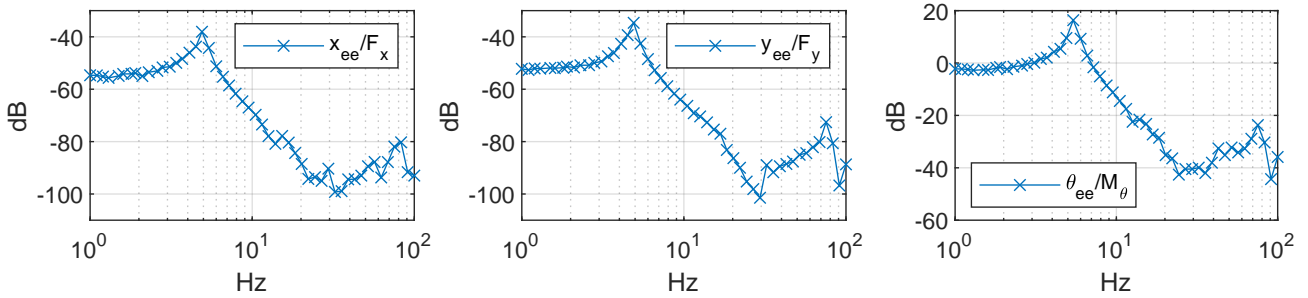


Figure 3: Diagonal parts of the experimental Bode magnitude plot of the system in the neutral configuration.

Although this model can evaluate the natural frequencies throughout the manipulator workspace quite efficiently, a system level optimisation with many (geometric) parameters would still be quite involved. Instead, the design is optimised in parts. At first the length  $L$  and angle  $\theta$  of the cartwheel flexure, see Figure 1(c), are optimised for a high compliance of the in-plane rotation and a high parasitic natural frequency. Next the geometry of the butterfly hinges is determined. Finally, the arm lengths are chosen to obtain acceptable rotations for the shoulder joint.

### 3 Numerical and experimental results

The dynamic properties of the complete manipulator are analysed with the advanced flexible multibody model. Figure 2 shows the relevant natural frequencies and mode shapes in the neutral configuration. The natural frequencies of both in-plane translational modes are identical because of the symmetry and are low (3.6 Hz) which confirms the desired high compliance. The natural frequency of the in-plane rotational mode is acceptable as well (7.9 Hz). Next eight modes with a natural frequency of about 53 Hz represent internal modes in the butterfly flexures. For the prototype manipulator these modes are ignored. The next mode 12 is an out-of-plane vibration of the end effector and is expected to be the performance limiting first parasitic mode. Its natural frequency of 76 Hz is about an order larger than the three lowest natural frequencies, which is an acceptable result. The variation of the natural frequencies throughout the workspace will be detailed in the full paper.

Figure 3 shows the diagonal parts of the experimentally obtained Bode magnitude plot of the system in the neutral configuration. It is obtained by exciting the system with multi-sine signals in the frequency range from 1 Hz to 100 Hz. The lowest translational frequencies appear to be somewhat higher than expected, which could to some extent be caused by the stiffness of the flexible coupling between actuator and upper arm. The internal mode of the butterfly flexures is not visible in the responses and hence safely ignored. The resonance frequency near 80 Hz agrees well with the expected frequency of the first relevant parasitic mode.

### 4 Conclusion

This paper shows a first design of a planar 3-DOF 4RRR parallel manipulator with redundant actuation and compliant joints. This first prototype shows dynamic behaviour in agreement with numerical expectations. It confirms that so far the design methodology and modelling are applicable and can be used to further optimise the system.

### References

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