Partitioning of Recursive Multibody Dynamics

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EXTENDED ABSTRACT

1 Introduction

Recursive multibody methods are some of the most efficient computational methods for multibody dynamics [1, 2]. The normal Newton-Euler form of the recursive equations of motion of a serial chain multibody system has the forms:

$$\begin{aligned} \underline{\mathcal{V}}(k) &= \underline{\phi}^*(k+1,k)\underline{\mathcal{V}}(k+1) + \underline{H}^*(k)\underline{\beta}(k) \\ \underline{\alpha}(k) &= \underline{\phi}^*(k+1,k)\underline{\alpha}(k+1) + \underline{H}^*(k)\dot{\underline{\beta}}(k) + \underline{\mathfrak{a}}(k) \\ \underline{\mathfrak{f}}(k) &= \underline{\phi}(k,k-1)\underline{\mathfrak{f}}(k-1) + \underline{M}(k)\underline{\alpha}(k) + \underline{\mathfrak{b}}(k) \end{aligned} \tag{1}$$

In the above, k denotes the body index, with body $k+1$ being inboard of body k. $\underline{\mathcal{V}}(k)$ denotes the spatial velocity of the k^{th} body, $\underline{\alpha}(k)$ denotes the spatial velocity of the k^{th} body, $\underline{\mathfrak{f}}(k)$ denotes the spatial inter-body force between the k^{th} and $(k+1)^{th}$ bodies, $\underline{H}^*(k)$ joint map matrix for the k^{th} body, $\underline{\beta}(k)$ the generalized velocities for the k^{th} body, and body, $\underline{M}(k)$ the spatial inertia of the k^{th} body, $\underline{\mathfrak{a}}(k)$ the spatial Coriolis acceleration and $\underline{\mathfrak{b}}(k)$ the spatial gyroscopic force. Eq. 1 establishes the recursive structure by defining the relationship of key quantities between child and parent bodies. These form of the equations of motion is quite general and applies to systems with rigid bodies, flexible bodies, flexible joints and even closed-chain systems using constraint embedding [2]. In this paper, our notational convention will be to use underline for regular dynamics symbols such as $\underline{\phi}$ above to distinguish them from the *partitioned* quantities used below.

In this paper we will examine conditions under which we can introduce an intermediate fictitious k' body in between the physical k and $k+1$ bodies and allocated a subset of the $\underline{\beta}(k)$ generalized velocities for the k^{th} body to the new k' body and leave only the remainder with the original k^{th} body. That is, we partition the generalized velocities as

$$\underline{\beta}(k) = \begin{bmatrix} \beta(k'') \\ \beta(k') \end{bmatrix} \tag{2}$$

Introduction of this fictitious body also as the effect of partitioning the equations of motion as follows:

$$\begin{aligned} \mathcal{V}(k') &= \phi^*(k+1,k')\mathcal{V}(k+1) + H^*(k')\beta(k') \\ \mathcal{V}(k) &= \phi^*(k',k)\mathcal{V}(k') + H^*(k'')\beta(k'') \\ \\ \alpha(k') &= \phi^*(k+1,k')\alpha(k+1) + H^*(k')\dot{\beta}(k') + \mathfrak{a}(k') \\ \alpha(k) &= \phi^*(k',k)\alpha(k') + H^*(k'')\dot{\beta}(k'') + \mathfrak{a}(k) \\ \\ \mathfrak{f}(k') &= \phi(k',k)\mathfrak{f}(k) + M(k')\alpha(k') + \mathfrak{b}(k') \\ \mathfrak{f}(k+1) &= \phi(k+1,k')\mathfrak{f}(k') + M(k+1)\alpha(k+1) + \mathfrak{b}(k+1) \end{aligned} \tag{3}$$

While this partitioning increases the number of recursion steps, it also reduces the dimensionality of the individual steps. The reduction in size can be computationally beneficial when the size of the generalized velocities is large such as for flexible bodies.

In this paper, we examine such partitioned dynamics, and derive the conditions under which the k' pseudo-body can be added such that the regular equations of motion in Eq. 1 remain equivalent to the partitioned ones in Eq. 3, or more specifically where the partitioned values at the k^{th} body agree with the regular ones. The equivalence of the equations of motion represented by Eq. 1 and Eq. 3, defined by the expressions,

$$\underline{\mathcal{V}}(k) = \mathcal{V}(k), \qquad \underline{\alpha}(k) = \alpha(k), \qquad \underline{\mathfrak{f}}(k) = \mathfrak{f}(k), \tag{4}$$

We show that the equivalence holds if the following conditions are satisfied:

$$
\begin{aligned}
\underline{\phi}(k+1,k) &= \phi(k+1,k')\ \phi(k',k) \\
\underline{H}(k) &= \begin{bmatrix} H(k'') \\ H_C(k) \end{bmatrix} \quad \text{where} \quad H_C(k) \triangleq H(k')\phi(k',k) \\
\underline{\mathfrak{a}}(k) &= \mathfrak{a}(k) + \phi^*(k',k)\mathfrak{a}(k') \\
\underline{M}(k) &= M(k), \qquad M(k') = 0 \quad \text{and} \quad \mathfrak{b}(k') = 0
\end{aligned}
\tag{5}
$$

These equivalence conditions imply that the ficititious k' body is mass-less. Thus the partitioning process consists of introducing a *massless* k' body between the $k+1$ and k bodies, and assigning some of the generalized velocities for the k^{th} body to the k' body and the remaining to the k^{th} body. While aggregating bodies is always possible, such decoupling via massless bodies is not always possible and requires additional underlying structure listed in Eq. 5 for the equivalence to hold.

We will refer to this assumption as the *Massless body decoupling assumption* and it will be assumed to hold in the rest of this chapter. Examples of systems where such decoupling is possible are:

- Body hinges physically composed of a sequence of sub-hinges: Examples of these are *universal joints* and *gimbal* hinges, where the hinges are a sequence of single degree of freedom subhinges. The dynamics model can be formulated as one where there are massless bodies between these individual subhinges. One benefit of this decomposition is that while the $H^*(k)$ matrix for the original hinges varies as the coordinates change, the joint map matrix remains constant for the individual subhinges in the partitioned formulation. This case represents partitioning arising from the trivial partitioning of the $H(k)$ matrix and the generalized velocities.
- Multi-dof translation hinges - such as planar hinges - can also be decomposed into a sequence of single degree of freedom subhinges. This is also a fall back to the trivial case partitioning the $H(k)$ matrix and the generalized velocities.
- Flexible bodies have generalized velocities that are a combination of ones from the hinge and the remaining from the body deformation. We will show that we can decouple these equations of motion by introducing a a massless pseudo-body between the hinge and deformation degrees of freedom. This partitioning reduces the size of $\mathcal{D}(k)$, and introduces sparsity in the partitioned $H(k)$ matrices that simplify and decouple algorithm computations. that of $\mathcal{V}(k)$.

In this paper we derive the articulated body inertia recursive forward dynamics algorithm for the partitioned form of the equations of motion. While the obtained solution obtained is the same as for the unpartitioned case, the partitioned form offers computational advantages. We prove the equivalence here. Furthermore we show the application of the partitioned form to recursive solution for flexible body systems.

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References

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