Multibody joint parameter estimation using an Augmented Extended Kalman filter

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EXTENDED ABSTRACT

1 Introduction

In recent years, state estimation techniques based on the Augmented Kalman filter [1] have been successfully used for nonlinear structural mechanics applications [2]. This approach assumes an Ordinary Differential Equation description of the underlying model used in the estimation algorithm. An Augmented Extended Kalman filter (AEKF) applicable to Differential Algebraic Equation (DAE)-described models has been introduced in [3]. This formulations is applicable to Multibody (MB) problems, as it enforces the holonomic MB constraints throughout the estimation process via projection of the Kalman update steps onto the constraints manifold. This contribution further extends the framework developed in [3] in order to perform estimation of parameters that are related to the holonomic constraints. The necessary theoretical framework and a numerical validation case are presented in the following sections.

2 Theory

In this work, MB systems are described in the continuous time domain using the following set of index-2 DAEs [4]:

$$\phi(\mathbf{q},\mathbf{v},\mu,\lambda,\mathbf{G})=\mathbf{0} \tag{1}$$

$$\mathbf{G}(\mathbf{q})\mathbf{v}=\mathbf{0} \tag{2}$$

$$\mathbf{g}(\mathbf{q},p)=\mathbf{0} \tag{3}$$

Where: $\mathbf{q},\mathbf{v}\in\mathbb{R}^{n_q}$ constitute the set of system generalized coordinates and first derivatives; $\mu,\lambda\in\mathbb{R}^{n_g}$ are, respectively, the two sets of Lagrange multipliers for position level and velocity levels constraints; $\mathbf{g}\in\mathbb{R}^{n_g}$ is the set of holonomic algebraic constraint equations, with $\mathbf{G}\in\mathbb{R}^{n_g\times n_q}$ its Jacobian with respect to $\mathbf{q}$; ϕ is the function that describes the equation of motion of a MB system in a condensed form; p represents a parameter that influences the constraint function $\mathbf{g}$. Accurate estimation of its value is the objective of this work. To accomplish this task using an AEKF, it is common to extend the set of DAEs (1-3) with a random walk model to represent the dynamics of the unknown parameter to be estimated:

$$\dot{p}=0 \tag{4}$$

The estimator used in this work is derived from the well-known AEKF formulation. The state vector is defined as $\mathbf{x}=[\mathbf{q}^T \quad \mathbf{v}^T \quad p]^T$. Expressing the set of equations (1-4) in discrete-time as $\mathbf{f_d}$, using k as timestep index, the state-space formulation becomes:

$$\mathbf{f_d}(\mathbf{x}_{k+1},\mathbf{x}_k,\gamma_{k+1})+\omega_{k+1}=\mathbf{0} \tag{5}$$

$$\mathbf{y}_{k+1}=\mathbf{h}(\mathbf{x}_{k+1})+\varepsilon_{k+1} \tag{6}$$

where $\mathbf{y}\in\mathbb{R}^{n_y}$ is the measurement signal; $\mathbf{h}$ is the measurement function; ω and ε are zero-mean Gaussian noises with known diagonal covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ that represent, respectively, uncertainties on both the system dynamics and the measurement equation. The estimator provides an estimate of the first two statistical moments of $\mathbf{x}$ given the sequence of measurements $\{\mathbf{y_k}\}$. The first moment is the mean value of the estimate, referred to as $\hat{\mathbf{x}}$, the second moment is the state error covariance matrix $\mathbf{P}$. At each time step, the AEKF used in this work performs the following actions:

1. **Prediction:** via integrations of the equations of motion, and propagation of the error covariance matrix using the plant error covariance matrix $\mathbf{Q}$, the algorithm provides a predicted estimate of $\hat{\mathbf{x}}^-_{k+1}$, $\mathbf{P}^-_{k+1}$.
2. **Correction:** using the Kalman update procedure, $\hat{\mathbf{x}}$, $\mathbf{P}$ are corrected as to decrease the mismatch between the measurement $\mathbf{y}_{k+1}$ and the prediction $\mathbf{h}(\hat{\mathbf{x}}_{k+1})$, taking into account the measurement noise covariance matrix $\mathbf{R}$. The end results are the corrected estimates $\hat{\mathbf{x}}^+_{k+1}$, $\mathbf{P}^+_{k+1}$.
3. **Constraint enforcement:** to ensure that the post-correction estimate is constraint-compliant, $\hat{\mathbf{x}}^+_{k+1}$ is projected onto the holonomic constraint manifold by solving the following constrained optimization problem:

$$\min_{\tilde{\mathbf{x}}_{k+1}} \quad (\tilde{\mathbf{x}}_{k+1}-\hat{\mathbf{x}}^+_{k+1})^T(\tilde{\mathbf{x}}_{k+1}-\hat{\mathbf{x}}^+_{k+1}) \qquad \text{s.t.} \qquad \mathbf{g}(\tilde{\mathbf{x}}_{\mathbf{k+1}})=\mathbf{0} \tag{7}$$

Repeating these actions at each timestep, the end result is a sequence of estimates $\{\tilde{\mathbf{x}}_k\}$ that is compliant with the set of constraints applied to the MB system.

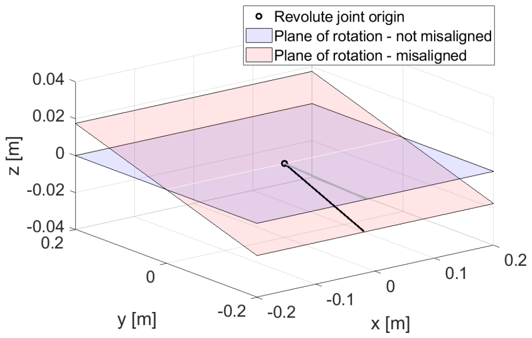
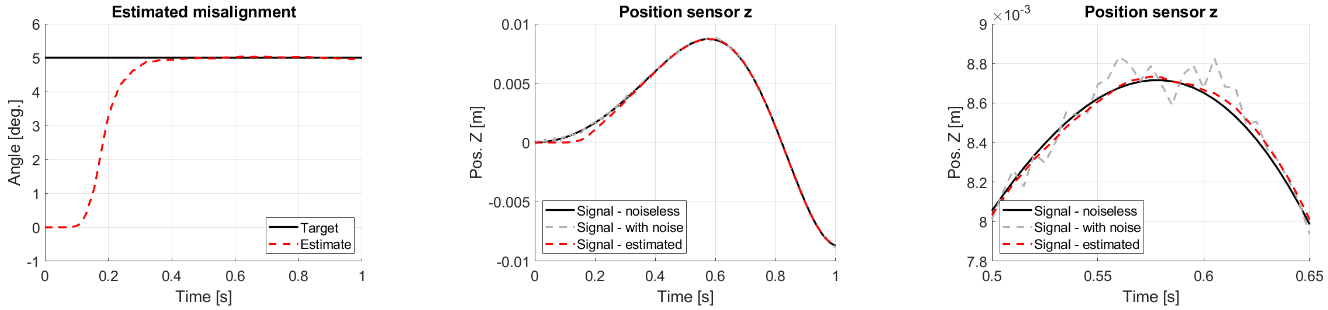


Figure 1: Visualization of the system used in the numerical validation.



(a) Angular misalignment estimate.

(b) Position sensor signal.

(c) Position sensor signal (zoom).

Figure 2: Estimation results of the validation.

3 Numerical Validation

The methodology has been validated with a virtual experiment. The MB system under exam consists of a rigid pendulum connected via revolute joint to the ground. The rotational axis of this joint is at an angle of 5°with respect to the world frame's z-axis. A visualization is provided in Figure 1. This system is used as reference to obtain position measurements that are going to be used in the estimator. White noise is added to the signal. The estimator uses a system that is equal to the reference one, but whose plane of rotation corresponds to the xy plane. The objective of the AEKF is the accurate estimation of the misalignment angle p. The results are visible in Figure 2: both the misalignment angle and the position of the pendulum are correctly estimated.

4 Conclusions

In this work, an AEKF formulation suitable for MB problems has been extended for the purposes of joint parameter estimation, and validated with a numerical experiment in which the rotational axis of a revolute joint has been corrected starting from noisy measurements coming from a reference system.

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