Modelling the inelastic constitutive behaviour of multi-layer spiral strands: comparison of hysteresis operator approach to rheological model

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EXTENDED ABSTRACT

The simulation of inelastic effects in flexible slender technical devices, such as cables or endoscopes, has become of increasing interest in the past years. Different approaches have been considered depending on the effects relevant for the specific application. Cables used in automotive applications and multi-layer spiral strands have in common that frictional interactions between elementary wires subjected to tensile force and bending loading result in nonlinear dissipative behaviour, i.e. hysteresis in the stress-strain relation. In our contribution, we compare a multiscale approach which includes modelling the interactions between wires including friction, with a data-based technique which uses hysteresis data determined on the macroscale to identify a hysteresis operator model. The data may stem from real experiments as shown in [1] or virtual experiments as done in the present work.

The first method is based on a computational homogenisation procedure to accurately characterise the nonlinear response of spiral strands. By using 1D beam elements in both micro- and macro-scale, homogenisation is performed along the axial direction of a representative volume element (RVE), leading to expressing a boundary value problem, driven in a mixed manner by either strains or resulting forces or moments. The boundary value problem on the RVE is solved using an implicit finite element solver for finite strain, considering all frictional contact interactions. The solution of the RVE boundary value problem is then used as an input for a rheological model capable of predicting the nonlinear biaxial response of spiral strands subjected to a variable tensile force (Fig. 1). The proposed model for an m-layer spiral strand consists of one elastic spring and m plastic springs acting in series. One of the main advantages of the proposed model is that parameters can easily be extracted from several monotonic uniaxial bendings under constant tensile force.

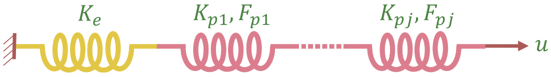

Figure 1: The equivalent spring system to a multi-layer spiral strand.

The second method is a data-based approach which makes use of mathematical tools called hysteresis operators. As shown in [2], hysteresis operators are a well-studied topic with a variety of applications, mainly hysteresis effects arising from electric and magnetic phenomena. The Prandtl-Ishlinskii (P-I) operator P plays a relevant role in modelling the input-output relation in phenomena showing hysteretic behaviour and can be expressed as a superposition of elementary stop operators S_r multiplied by a suitable weight function $\omega_{\mathrm{P}}(r)$, which is assumed to vanish for large values of r. We aim at expressing the bending moment (M_B) vs. bending curvature (K_B) in terms of P-I operator as a discretised version of

$$M_B(t) = \mathrm{P}_r[K_B](t) = \int_0^{+\infty} \omega_{\mathrm{P}}(r) S_r[K_B](t)\mathrm{d}r. \tag{1}$$

The stop operator S_r can be defined recursively, and can itself be equivalently formulated as a simple rheological model. By superimposing different elementary stop operators, one is able to model more complex hysteretic effects, also taking into account the history of the process. Fig. 2 shows an example of input-output relation of the stop operator with threshold $r = 2$ applied to a sinusoidal signal.

In this contribution, we first make use of bending moment vs. bending curvature virtual data generated by means of the described homogenisation method to train a hysteresis model which replicates the constitutive behaviour of a given (virtual) spiral strand. We then highlight the relationship between the weight function ω_{P} and the thresholds r in a discrete framework, respectively with the spring stiffnesses K_e, K_{pj} and the kinematic hardening parameters F_{pj} for $j = 1,\ldots,m$, where m is the number of layers of the spiral strand. This allows us to find a physical interpretation of the P-I operator when modelling nonlinearities arising from friction between wires in spiral strands (Fig. 1). We compute the discrete weights of a suitable P-I operator via a

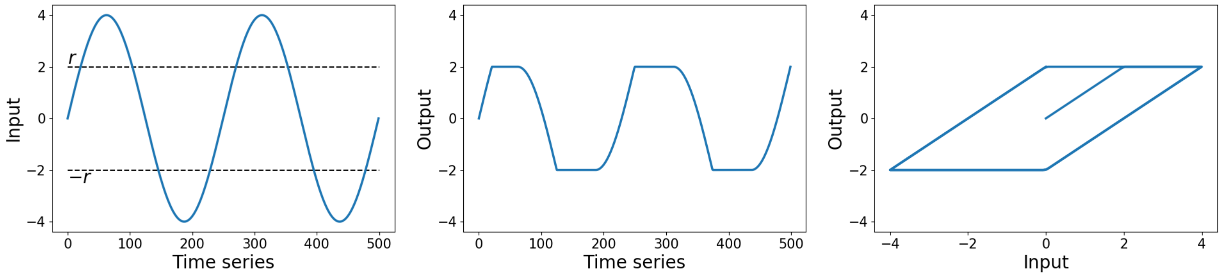

Figure 2: Example of stop operator. *Left*: Sinusoidal input function, the dashed lines represent the threshold values $\pm r = \pm 2$. *Centre*: Output function of the stop operator. *Right*: Input-output diagram.

minimisation procedure and compare the results as shown in Fig. 3 *right*. The same procedure is applied to data coming from virtual experiments performed on different types of spiral strands with varying number of layers m.

In conclusion, the present work investigates the possibility to model inelastic effects in spiral strands arising from friction between wires in two different ways. While the first approach takes the inter-wire effects directly into account, the second one is a grey-box model strategy. This comparison leads to a better understanding and an explicit physical interpretation of the parameters of a specific class of hysteresis operator models.

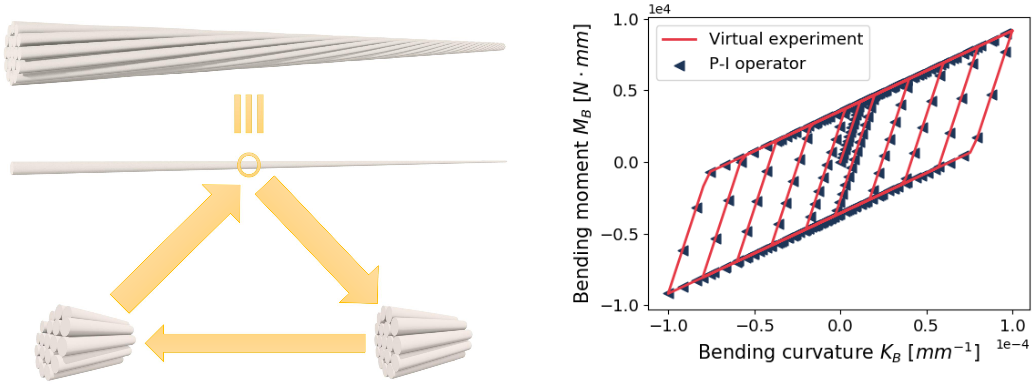

Figure 3: *Left*: The mixed stress-strain driven computational homogenisation framework. *Right*: Bending moment v. bending curvature diagram. Solid line: virtual experiment data obtained via the homogenisation technique, scattered plot: data reproduced by means of a hysteresis operator.

Acknowledgments

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