A Continuous Contact Force Model for Highly Damped Impacts of Arbitrary Material and Geometry

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EXTENDED ABSTRACT

1 Introduction

The most widely-used and accepted dissipative contact force model is that which was proposed by Hunt and Crossley in 1975. Equation (1) presents the Hunt-Crossley model in its equivalent dynamic form,

$$m\ddot{\delta} + \lambda\delta^n\dot{\delta} + K\delta^n = 0, \tag{1}$$

where δ is the relative deformation. Since this formulation, most of the proceeding work done on the model has been in the determination of λ. λ, the hysteresis damping coefficient, unlike the mass, m, and (sometimes) the stiffness, K, is not intrinsically understood, and as such has no analytically derived expression. Therefore, to compute λ, it is common practice to relate it to the coefficient of restitution, c_r. Many researchers have proposed expressions for λ in terms of c_r, done initially by Hunt-Crossley themselves and henceforth by Lankarani-Nikravesh, Flores et al., Hu-Guo, and others [1, 2, 3, 4]. These expressions, however, are all restricted to cases of low damping, and become quickly inaccurate where energy losses are high.

2 Problem description

One goal of this work is to indeed provide an improved expression for λ, one that accommodates cases of high damping. This work seeks similarly, however, to derive an expression for the system stiffness also. Although more intrinsically understood, analytical expressions for K are still limited and exist only for special cases, in the contact between two linear elastic spheres, for example [2]. For more complex contacts, K becomes another unknown parameter. If the stiffness, therefore, can be related to an empirical parameter in a similar way as λ is done to c_r, the general practicality of the Hunt-Crossley model would be greatly improved and its use far more variable. In this work, it is desired to reformulate the Hunt-Crossley equation, present a more accurate expression for λ that accommodates cases of high damping, and generate an empirical method for determining K.

3 Applied methods and results

3.1 Hysteresis damping ratio and impact natural frequency

Before determining expressions for λ and K, it is first useful to derive more fundamental analytic expressions of the dynamic equation. Like the traditional linear Kelvin-Voigt model, similar behavioral parameters can be derived for the Hunt-Crossley model, namely the hysteresis damping ratio and impact natural frequency. Through the nondimensionalization of equation (1) and subsequent manipulations, the two parameters are derived as follows:

$$\varepsilon = \frac{\lambda\dot{\delta}_0}{K}, \tag{2}$$

$$\omega_i = \left[\frac{K\dot{\delta}_0^{\,n-1}}{m}\right]^{\frac{1}{n+1}}. \tag{3}$$

ε is the hysteresis damping ratio, ω_i is the impact natural frequency, and $\dot{\delta}_0$ is the relative velocity before impact. Analytical expressions for damping ratio and natural frequency are useful in that they enable one to better understand which parameters affect damping and impact duration. With these parameters, a new dimensional form of equation (1) is derived:

$$\ddot{\delta} + \frac{\omega_i^{n+1}}{\dot{\delta}_0^{\,n}}\delta^n(\varepsilon\dot{\delta} + \dot{\delta}_0) = 0. \tag{4}$$

3.2 Expression for hysteresis damping coefficient

The current limiting step in determining an accurate expression for λ in terms of c_r is in the velocity-deformation relationship, $\dot{\delta}(\delta)$. The velocity throughout impact can be split into the compression ($\dot{\delta}_c$) and restitution ($\dot{\delta}_r$) phases, and, when $\lambda = 0$ (no damping), there are exact analytical solutions for them. However, when $\lambda \neq 0$, there are no such solutions, and approximations must be used. The accuracy of these approximations is crucial in the accuracy of the derived $\lambda(c_r)$ function. The best approximations used currently, used by Hu-Guo, Flores et al., and others, are those of the $\lambda = 0$ analytical solutions with c_r multiplied to the restitution function [3, 4]. The final $\lambda(c_r)$ expressions derived from these approximations fair well for $c_r > 0.6$ or so, but better approximations can be made to accommodate c_r approaching zero. The approximations in this work are as follow:

$$\begin{aligned} \dot{\delta}_c &= \dot{\delta}_0 \left[1 - \left(\frac{\delta}{\delta_m} \right)^{n+1} \right]^{\frac{1}{2}+p}, \\ \dot{\delta}_r &= -c_r \dot{\delta}_0 \left[1 - \left(\frac{\delta}{\delta_m} \right)^{n+1} \right]^{\frac{1}{2}-p}. \end{aligned} \tag{5}$$

Here, $p = \alpha(e^{-\beta c_r} - e^{-\beta})$, where α and β are constants. This introduced p function accommodates how the velocity functions deviate from a perfect square root with respect to decreasing restitution coefficient, accommodating cases of high damping. The value of p ranges from 0 to $1/2$. With these new velocity approximations, the hysteresis damping coefficient is derived using the traditional energy balance method. The result in terms of the hysteresis damping ratio is presented:

$$\varepsilon(c_r) = \frac{(3 + 2\alpha(e^{-\beta c_r} - e^{-\beta}))(1 - c_r^2)}{2c_r} \left(\frac{3 + 2\alpha(e^{-\beta c_r} - e^{-\beta})}{3 - 2\alpha(e^{-\beta c_r} - e^{-\beta})} + c_r \right)^{-1}. \tag{6}$$

Fitting the constants α and β against the numerical solution for $\varepsilon(c_r)$ gives $\alpha = 1.381$ and $\beta = 0.451$. On the range of restitution coefficients to 0.01 ($0.01 \leq c_r \leq 1.0$), the relative error in equation (6) to the numerical solution is 0.062% compared to the Hu-Guo solution of 47.768% and the Flores et al. solution of 57.617% [3, 4].

3.3 Expression for impact natural frequency (system stiffness)

With an expression for ε, equation (4) can now be used to model an impact if the system stiffness is known. If K is unknown, however, ω_i is also unknown and the model is still unusable. The derived expression for the impact natural frequency in section 3.1 is now useful for it can be related to another empirical parameter, that being the impact time, Δt. Working with the dimensionless form of equation (1), an expression for the dimensionless impact time, $\Delta\tau$, is derived through a step-by-step fitting approach:

$$\Delta\tau(\varepsilon, n) = ae^{-b\varepsilon} + (c - de^{-fn})\varepsilon + (gn - h)e^{-in} - jn + k. \tag{7}$$

Here, e is Euler's Number and a, b, c, d, f, g, h, i, j, and k are numerical constants. Through surface fitting equation (7) to the numerical solution, the constants produced are $a = 0.180$, $b = 0.880$, $c = 0.853$, $d = 1.018$, $f = 0.461$, $g = 1.434$, $h = 0.864$, $i = 0.771$, $j = 0.023$, and $k = 2.695$. The relative error in equation (7) to the numerical solution is is 0.237%. Converting to dimensional form, an expression for ω_i in terms of Δt is derived (which can be used with equation (3) to determine K):

$$\omega_i(\varepsilon, n, \Delta t) = \frac{1}{\Delta t} \left[ae^{-b\varepsilon} + (c - de^{-fn})\varepsilon + (gn - h)e^{-in} - jn + k \right]. \tag{8}$$

4 Conclusion

In this work, a new form of the Hunt-Crossley contact force model is presented with derived parameters ε and ω_i. The formulation of these two parameters is useful in that they give insight into behavioral aspects of Hunt-Crossley not previously considered. From new approximations of the compression and restitution phase velocity functions that take into account profile changes, a more accurate expression for the hysteresis damping coefficient/ratio is presented to accommodate cases of high damping. Finally, an expression for the impact natural frequency is produced, for general ε and n, that relates it to impact time, a first time method for allowing K, alongside λ, to be determined empirically.

References

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