Damping Coefficient for Impacts with Residual Deformation at the Time of Separation

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EXTENDED ABSTRACT

Many engineering systems experience impact due to joint clearances, joint locking, and/or collisions between different components. Various research efforts have been conducted in this area to generate appropriate models when a multibody system experiences impacts [2, 7]. These models are generally created based on two different approaches: piecewise method and continuous approach. In the continuous method which is the focus of this paper, contacting bodies are allowed to experience penetration during the period of impact, resulting in the continuous change in the positions and velocities. Therefore, contact force f_c is active during the period of impact. For instance, for a direct central impact of spheres of masses m_i and m_j in Fig.1, equations of motion for each body during the period of contact can be constructed as

$$m_i\ddot{\delta}_i = -f_c, \;\; m_j\ddot{\delta}_j = f_c, \tag{1}$$

where δ_i and δ_j denote the coordinates of the mass centers of spheres i and j, respectively in the direction of $\mathbf{n}$ normal to the surface of contact. Defining the penetration $\delta = \delta_i - \delta_j$ and effective mass $m_{eff} = \frac{m_i m_j}{m_i+m_j}$ [8], the following Ordinary Differential Equation (ODE) governs the dynamics of the penetration

$$m_{eff}\ddot{\delta} = -f_c, \qquad \delta(0) = {}^{-}\delta = 0\,, \qquad \dot{\delta}(0) = {}^{-}\dot{\delta}, \tag{2}$$

where ${}^{-}\delta$ and ${}^{-}\dot{\delta}$ are the indentation and indentation speed immediately before the impact. The contact force f_c is mainly represented by a logical point-to-point spring-damper element [3] as

$$f_c = f_d + k\delta^n, \tag{3}$$

where f_d is the damping force and $k\delta^n$ represents the spring force. The values of k and n depend on the geometry and/or the material properties of the contacting surfaces [6]. The main challenge in this model is how to express the damping force. For example, a linear damper in the form of

$$f_d = c\dot{\delta} \tag{4}$$

has been used in Kelvin-Voigt viscoelastic model [3]. The dotted curve in Fig. 2 shows the behavior of this contact force versus penetration. It is observed that a non-zero compressive contact force exists at point A which corresponds to ${}^{-}t$, immediately before the impact, while point B represents the separation of spheres. It is clear that using this model beyond point B generates a non-physical tensile force. Using an optimization approach, the damping coefficient c in Eq. (4) has been expressed as [9]

$$c = A\,(e^B - 1)\,\left[k\,({}^{-}\dot{\delta})^{n-1}\,(m_{eff})^n\right]^{\frac{1}{n+1}}, \tag{5}$$

$$A = 0.3331\,n^4 - 1.48\,n^3 + 3.077\,n^2 - 2.306\,n + 1.794, \qquad B = 1.285\,n^{0.2553} - 1.725, \tag{6}$$

such that the model does not generate the non-physical behavior beyond point B. In this expression, e represents the coefficient of restitution defined as $e = -\frac{{}^{+}\dot{\delta}}{{}^{-}\dot{\delta}}$, where ${}^{+}\dot{\delta}$ is the indentation speed immediately after the impact.

To account for non-physical tensile contact force beyond point B, Hunt and Crossley [5] proposed the damping force in the form of

$$f_d = \mu\dot{\delta}\delta^n, \tag{7}$$

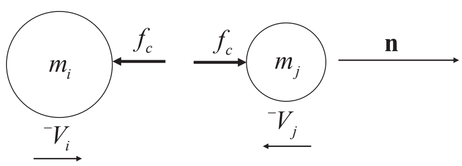

Figure 1: Direct-central impact of two spheres

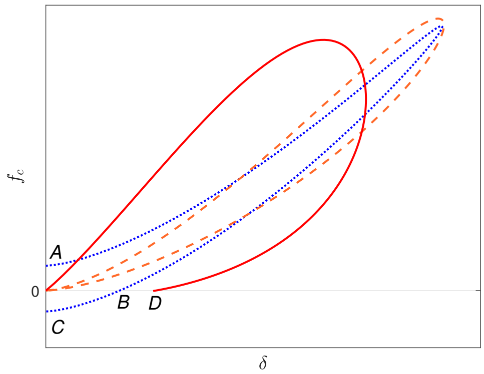

Figure 2: Behavior of the contact force versus penetration for different force models

where μ is called “hysteresis damping factor”. The dashed curve in Fig. 2 shows that colliding bodies experience zero contact force and penetration at both beginning and end of contact. Various research efforts generated expression for the hysteresis damping factor of Eq. (7) as [6, 4, 1]

$$\mu = \frac{3k(1-e^2)}{4(-\dot{\delta})}, \quad \mu = \frac{3k(1-e)}{2e(-\dot{\delta})}, \quad \mu = \frac{8k(1-e)}{5e(-\dot{\delta})}. \tag{8}$$

As it is observed in Fig. 2, the dotted curve which is based on the damping model in Eq. (4) generates a non-zero force at the beginning of impact which may not be realistic according to some reports [10]. On the other hand, the Hunt-Crossley model with the damping force of Eq. (7) cannot represent impacts with non-zero deformation at the time of separation. In this paper, we present a new model in the form of

$$f_c = \mu\dot{\delta}\delta + k\delta^n, \tag{9}$$

which accounts for both of the shortcomings of the two models explained previously. The new model which is schematically shown in the solid curve in Fig. 2 is able to generate a zero contact force at the beginning of impact, while capable of addressing the situations with residual deformation at the end of impact. This will be accompanied by generating analytical and computational framework to find appropriate hysteresis damping factor. Finally, the new method is compared with already-established techniques available in literature.

References

[1] P. Flores, M. Machado, M.T. Silva, J.M. Martins: On the continuous contact force models for soft materials in multibody dynamics. Multibody System Dynamics, 25:357–375, 2011.

[2] G. Gilardi, I. Sharf: Literature survey of contact dynamics modelling. Mechanism and Machine Theory, 37:1213–1239, 2002.

[3] W. Goldsmith. Impact: the theory and physical behaviour of colliding solids. Courier Corporation, 2001.

[4] S. Hu, X. Guo: A dissipative contact force model for impact analysis in multibody dynamics. Multibody System Dynamics, 35:131–151, 2015.

[5] K. Hunt, F.R.E. Crossley: Coefficient of restitution interpreted as damping in vibroimpact. Journal of Applied Mechanics, 42:440–445, 1975.

[6] H.M. Lankarani, P.E. Nikravesh: Continuous contact force models for impact analysis in multibody systems. Nonlinear Dynamics, 5:193–207, 1994.

[7] M. Machado, P. Moreira, P. Flores, H.M. Lankarani: Compliant contact force models in multibody dynamics: Evolution of the Hertz contact theory. Mechanism and Machine Theory, 53:99–121, 2012.

[8] P. E. Nikravesh, M. Poursina: Determination of effective mass for continuous contact models in multibody dynamics. Multibody System Dynamics, 1–21, 2022.

[9] M. Poursina, P.E. Nikravesh: Optimal damping coefficient for a class of continuous contact models. Multibody System Dynamics, 50:169–188, 2020.

[10] W. Wang, X. Hua, X. Wang, Z. Chen, G. Song: Advanced impact force model for low-speed pounding between viscoelastic materials and steel. Journal of Engineering Mechanics, 143: 04017,139, 2017.