Simulation of spherical rigid bodies subject to friction with multiple impacts

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EXTENDED ABSTRACT

1 Introduction

The characterization of mechanical systems with multiple impacts between spherical rigid bodies under friction contact is a challenging research topic that allows the simulation of different kinds of problems such as the typical billiard break, the Newton's cradle toy and the Bernoulli's problems, among others [10]. When a multibody system is subjected to impacts, two scenarios can arise: (i) **single impacts**, when the bodies are in contact at a single point and the impact occurs at this point, and (ii) **multiple impacts**, when there are several points in contact and the impact occurs simultaneously in some of them [11].

There are two main approaches for numerically solving impact problems. The first group assumes that the impact time duration is very small and, the second group describes the impact process as instantaneous. Then, the first group can be even subdivided into the first-order models, which are based on Darboux [5] approach, and the second-order models where some kind of spring-dashpot models are used. Liu and Brogliato extended the first-order formulation to study granular chain problems [9]. In the second group, the impact process is modeled by using the classical Newton impact law which relates the pre-impact velocity and the post-impact velocity by a restitution coefficient. Another alternative is using the Poisson impact law, which assumes a decomposition of the impact process into a compression phase and expansion phase and involves a coefficient of restitution [11]. The adoption of one of these local laws for modeling multiple impacts is a natural and convenient choice. However, additional complications appear in the formulations when frictional effects are considered because a nonlinear friction law such as the Coulomb's law is needed.

In this work, we introduce a new methodology for the simulation of frictional multiple impact collisions between spherical rigid bodies in the framework of nonsmooth contact dynamics and the finite element method for problems with large rotation. The proposal is an extension of the frictionless multiple impacts algorithm to the frictional case based on the Newton's impact law presented by Cosimo *et al.*[4]. The algorithm defines a sequence of impact problems on a vanishing time interval and the active set of each velocity-level sub-problem is redefined in the normal and tangential direction, in such a way that closed contacts with zero preimpact velocity are considered inactive.

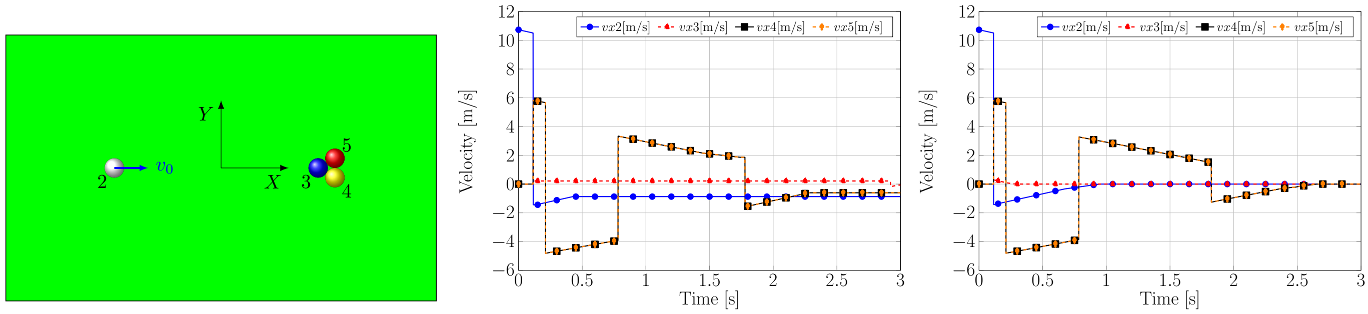
The sphere-plane and the sphere-sphere contact elements formulated by Cavalieri *et al.* [3] are used in the numerical examples. These elements are extended to manage sliding, rolling and drilling friction effects. The contact problem is solved using an augmented Lagrangian formulation as proposed by Alart and Curnier [1] in quasi-static cases and applied later by Galvez *et al.* [6] to dynamic problems with friction. The total motion is directly referred to an inertial frame for the kinematic description of the rigid bodies [7]. Finally, the equations of motion are integrated using the nonsmooth generalized-α time integration scheme [4].

2 Numerical Example

The numerical example was initially proposed by Gismeros *et al.* [8]. It consists in a typical billiard break which allows to study the capacity of the algorithm to solve problems with multiple impacts with and without friction. According to [8], a white ball labeled 2 with a speed of $v_x = 10.792$ m/s hits three balls labeled 3, 4 and 5 that are in contact between them and at rest (see Fig. 1-a). The four balls have a radius $R = 0.028575$ m, a weight $mg = 1.666$ N and an inertia $I = 0.000055$ kg m^2. The table has a length of 2.54 m and a width of 1.27 m [2]. The values of the friction coefficient μ and the normal restitution coefficient e_N are 0.2 and 0, respectively, for the contact between the spheres and the table. For the contact between spheres, the coefficients are $\mu = 0.06$ and $e_N = 0.93$ while for the contact between the spheres and the edges of the table, $\mu = 0$ and $e_N = 0.85$, and in all contact points a tangential restitution coefficient $e_T = 0$ is imposed.

Two cases are analyzed: in the first one, the rolling resistance between the spheres and the plane is neglected, while in the second one a rolling resistance radius $\rho = 0.005$ m is adopted. The total simulation time was 3 s with a time step of 1×10^{-3} s. In the first case, the cue ball starts with a velocity of 10.729 m/s and null rolling velocity, and impacts the balls at a slightly lower velocity due to the sliding friction between the ball and the plane, see Fig. 1-b. After the multiple impacts, ball 3 moves forward with a low velocity. As it can be seen, once the balls are in pure rolling, their velocity remains constant (Fig. 1-b). The second case is similar to the first, however, the balls reach the rest condition due to the action of the rolling resistance, see Fig. 1-c.

The results show that the proposed methodology does not present any penetration between bodies in contact, contrary to what happened using the methodology given by Gismeros *et al.* [8] based on the penalty approach. Furthermore, the computing time was reduced from 25000 s, as demanded in the case of Gismeros *et al* to only 40 s.



(a) Billiard break configuration. (b) With sliding friction. (c) With sliding and rolling friction.

Figure 1: Results obtained for the billiard break.

3 Conclusions

A new methodology for handling simultaneous multiple impacts with friction effects between spherical rigid bodies was presented. The algorithm is based on the frictionless proposal by Cosimo *et al.* [4] in which the Newton impact law is sequentially applied by assuming instantaneous local impact times. The studied example demonstrated that the proposed methodology keeps a low computational cost compared with the classical penalty approaches. Furthermore, the strategy does not require any intervention of the user or any topological analysis for defining the sequence for processing the multiple impacts.

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