Sparse Identification of Unknown Equation of Motion Terms Associated with Complex Joint Phenomena in Multibody System Dynamics

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EXTENDED ABSTRACT

1 Introduction and Problem Statement

Higher, faster, and further on the one hand, but also safer and more efficient on the other hand – this trend demands a great deal of today's and future engineers. In addition to this challenging maxim, economic aspects drive the push to reduce the time to market of products required to master today's challenges. This has resulted in society drifting away from physical testing and towards virtual prototyping.

Multibody system (MBS) dynamics simulations are powerful tools to realistically analyse real-world, multiple-component devices in their intended operating environment. However, the inherently non-linear governing equations are often partially unknown. This is especially true for the equation of motion (EOM) terms representing contact forces acting within joints. Not only is the effect of these forces on the dynamics not precisely known, but contact mechanisms, especially friction, are still poorly understood [1]. Contact dynamics is still one the most challenging areas in the engineering sciences [2].

The aforementioned problems may be addressed with data-driven approaches, especially in our era, marked by an abundance of data and computing power. There have been exciting advances in the field of data-driven science and engineering, which enable researchers to, e.g., obtain the analytical expressions of the governing equations of dynamical systems [3, 4]. However, these algorithms are often tested on academic problems with few degrees of freedom (DOFs) and/or simulated data, only. Also, it is often assumed that the governing equations are fully unknown, which might go beyond what is necessary, as, e.g., inertia forces are often known. So a more tailor-made strategy for MBSs would model analytically what can be modelled well and use data only for the effects that are unknown and/or inherently hard to model with classic mechanical approaches.

2 Identification of Analytical Expressions of Unknown Joint Forces

Conventionally the EOMs of MBSs are formulated as index-3 differential algebraic equations (DAEs), where joints are assumed to be ideal (geometrical constraints) and enforced using Lagrange multipliers. In reality, joints are not ideal but subjected to complex contact phenomena, such as friction and gap opening/closing. In this case, the EOMs may be written (in 1$^{\text{st}}$ order form) as

$$\dot{\boldsymbol{s}}(t) = \boldsymbol{f}^{\mathrm{k}}(\boldsymbol{s}(t), t) + \boldsymbol{f}^{\mathrm{u}}(\boldsymbol{s}(t), t) \tag{1}$$

with the states $\boldsymbol{s}(t) = \begin{bmatrix} \boldsymbol{q}(t)^\top & \dot{\boldsymbol{q}}(t)^\top \end{bmatrix}^\top$ including the generalized coordinates $\boldsymbol{q}(t)$ and velocities $\dot{\boldsymbol{q}}(t)$, and where the right hand side (RHS) of Eq. (1) is split into known and unknown EOM terms, i.e., $\boldsymbol{f}^{\mathrm{k}}(\boldsymbol{s}(t), t)$ and $\boldsymbol{f}^{\mathrm{u}}(\boldsymbol{s}(t), t)$, respectively.[1]

The unknown EOM terms may be approximated as a weighted sum of non-linear functions stored in a library $\boldsymbol{\lambda}$, i.e., [4]

$$f_s^{\mathrm{u}}(\boldsymbol{s}(t), t) \approx \boldsymbol{\xi}_s^\top \boldsymbol{\lambda}(\boldsymbol{s}(t), t) \quad \Leftrightarrow \quad \boldsymbol{f}^{\mathrm{u}}(\boldsymbol{s}(t), t) \approx \boldsymbol{\Xi}^\top \boldsymbol{\lambda}(\boldsymbol{s}(t), t), \tag{2}$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\xi}_1 & \dots & \boldsymbol{\xi}_S \end{bmatrix} \tag{3}$$

includes the weights $\boldsymbol{\xi}_s$ $(s = 1, \dots, S)$ for all S states and [2]

$$\boldsymbol{\lambda}(\boldsymbol{s}(t), t) = \begin{bmatrix} \lambda_1(\boldsymbol{s}(t), t) \\ \vdots \\ \lambda_L(\boldsymbol{s}(t), t) \end{bmatrix} \overset{\text{e.g.}}{=} \begin{bmatrix} 1 \\ \boldsymbol{s}(t)^{\circ n} \\ \vdots \\ \sin(\boldsymbol{s}(t)) \\ \vdots \end{bmatrix}. \tag{4}$$

If one now defines the snapshot matrix of the states

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{s}(t_1) & \dots & \boldsymbol{s}(t_M) \end{bmatrix} \quad \Rightarrow \quad \dot{\boldsymbol{S}} = \begin{bmatrix} \dot{\boldsymbol{s}}(t_1) & \dots & \dot{\boldsymbol{s}}(t_M) \end{bmatrix}, \tag{5}$$

[1] $\dot{\bullet}$ denotes differentiation with respect to (w.r.t.) time t.
[2] $\bullet^{\circ n}$ denotes the Hadamard/element-wise n^{th} (here $n > 0$) power of $\bullet$.

and of the known RHS

$$\boldsymbol{F}^{\mathrm{k}} = \left[\boldsymbol{f}^{\mathrm{k}}\left(\boldsymbol{s}\left(t_1\right), t_1\right) \quad \dots \quad \boldsymbol{f}^{\mathrm{k}}\left(\boldsymbol{s}\left(t_M\right), t_M\right)\right], \tag{6}$$

where M is the number of snapshots/measurements, one can write

$$\dot{\boldsymbol{S}} \approx \boldsymbol{F}^{\mathrm{k}} + \boldsymbol{\Xi}^{\top}\boldsymbol{\Lambda}, \tag{7}$$

where

$$\boldsymbol{\Lambda} = \left[\boldsymbol{\lambda}\left(\boldsymbol{s}\left(t_1\right), t_1\right) \quad \dots \quad \boldsymbol{\lambda}\left(\boldsymbol{s}\left(t_M\right), t_M\right)\right]. \tag{8}$$

To identify the analytical expressions of the unknown governing equation terms, the number of non-zero elements (nnze) in the weight matrix has to be minimized while, e.g., satisfying a norm of the residual of Eq. (7) up to a user-defined tolerance τ, i.e.,

$$\min \operatorname{nnze}\left(\boldsymbol{\Xi}\right) \text{ subject to } \left\|\dot{\boldsymbol{S}} - \boldsymbol{F}^{\mathrm{k}} - \boldsymbol{\Xi}^{\top}\boldsymbol{\Lambda}\right\| < \tau, \tag{9}$$

which is usually achieved with a sparsity-promoting algorithm, such as the sequential thresholded least-squares (STLS) [4] or least absolute shrinkage and selection operator (LASSO) [5] algorithm. Measures to ensure physicality of the solution may also be introduced such as enforcing passivity or satisfying Newton's third law.

3 Experimental Setup

In order to evaluate the robustness and practical applicability of the proposed technique, a physical crank-shaft-mechanism test bed is used. A closed-loop controlled motor drives the crank with a desired velocity profile, while a piezo-transducer measures the resulting reaction forces and torques. A series of motion-tracking, laser-based and acceleration sensors measure the dynamics of the system. This large range of measurements ensure that a broad range of resolutions, bandwidths and absolute accuracies are covered in order to provide the data-driven approach with the best possible chance of capturing the system physics. To support the data-driven approach, a numerical simulation receives the measured reaction forces at the motor and generates a prediction for the system response. The data-driven approach then merely has to bridge the gap from the numerical prediction to the measurements.

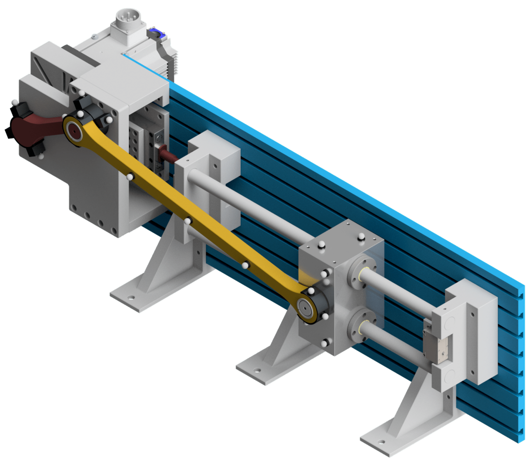

Figure 1: Digital mock-up of the crank-shaft-mechanism.

This single-degree of freedom (DOF) system has been designed to be modifiable to study the algorithm's performance for various types of data and dynamics. The contact surfaces of each joint can be swapped out or entire bodies can to modified to be more flexible. This sensitivity analysis is helpful to determine what type of problems this approach is best suited for.

4 Conclusions

In summary, the goal of this contribution is to use data-driven approaches to (i) advance the understanding of the physics of joint phenomena in MBSs, (ii) identify the analytical expressions of only the unknown joint contact forces, and (iii) test the algorithm of Sect. 2 with actual experimental data on complex mechanical systems, such as the test bed described in Sect. 3.

References

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