Warm-starting procedure involving penalties instead of constraints to find more optimal trajectories

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EXTENDED ABSTRACT

1 Introduction

Optimal control is often performed through a gradient descent allowing to find a local minimum satisfying the constraints upon convergence. However, for non-convex problems, it is rarely possible to know if the local minimum reached corresponds to the global one. To work around this limitation, it is possible to proceed through a multi-start approach [2] to explore a wider variety of local minima in the search for the global one. This multi-start is often done with random initialization. However, the low quality of a random initial starting point might affect the convergence of the optimal control problems (OCP). When the initial guess is far from an optimum, the interior point method (IPM) is often used due to its high convergence rate and its robustness to the initial guess. However, IPM might not be suitable to find minimum that are hidden in highly non-convex constrained domains due to its descent method [1]. Therefore, it would be advantageous to take benefits from the high convergence rate of the IPM method while getting around its drawbacks by starting the optimization at points beneath the constraint barriers. The objective of this study was to propose a method to generate initial guesses that are closer to an optimum and not necessarily positioned on the gradient's descent trajectory enclosed by the constraints. We hypothesized that this method would allow to find a relevant variety of local minima while being independent from the researcher's insights on the problem at hand. The suggested method was compared with traditional multi-start in terms of convergence rate, optimal cost, and computational time. A secondary objective was to assess the effect of the weight of the penalties and the maximum number of iterations during the first optimization on the optimal solutions.

2 Methods

The proposed workflow is composed of the following steps:

Initialization – Several random initial guesses are generated using a continuous uniform distribution for the position, velocity, and torque of the degrees of freedom (DoF). The random initialization allows to generate a variety of initial guesses that are independent from the researcher's knowledge of the problem. This increases the chances of finding different local minima.

First optimization – Some constraints of the original problem are replaced by penalties included in the cost function. This underconstrained OCP is easier to solve and less restrained by the IPM constraint barriers. Note that even if an optimal solution was found at this step, there would be no guarantee that the solution would be dynamically consistent and respect the sphere constraints. However, the solution is likely to be closer to an optimum and more dynamically consistent than the random initialization.

Second optimization – The solution from the first optimization is provided as the initial guess to the fully constrained OCP.

We applied our workflow to a multiple-shooting OCP implemented in Bioptim [3]. The OCP consisted in inverting a rigid pendulum without hitting constraint spheres along the way (Fig. 2). The passive pendulum was mounted on a cart which could translate on a rail. The objective of the OCP was to minimize the force needed to translate the cart and to minimize the time needed to complete the inversion.

For comparison, the OCP was solved 100 times with three different implementations. *i)* The problem was solved with the inter-shooting node continuity and the collision spheres included in the problem as constraints (***Constrained***). *ii)* Our workflow was applied with the continuity (***Continuity penalised***) included in the problem as a penalty of the following form:

$$\sum_{i=1}^{N_i-1} \sum_{k=1}^{N_k} (x^k_{i+1,j=0} - x^k_{i,j=N_j})^2 \qquad \begin{array}{l} \text{where } x^k \text{ is the } k^{th} \text{ state variable} \\ i \text{ is the number of the shooting node} \\ j \text{ is the number of the integration step} \end{array} \tag{1}$$

iii) It was also applied with the collision spheres (***Spheres penalised***) included in the problem as penalties of the following form:

$$\sum_{i=0}^{N_i} \sum_{l=1}^{4} \left| \begin{cases} (P_i^{marker} - P_{i,l}^{sphere}) - r^{sphere}, & \text{if negative,} \\ 0, & \text{otherwise} \end{cases} \right| \qquad \begin{array}{r} \text{where } P_l^{sphere} \text{ is the position of the } l^{th} \text{ sphere} \\ P^{marker} \text{ is the position of the pendulum} \\ r^{sphere} \text{ is the radius of the spheres} \end{array} \tag{2}$$

The convergence rate, optimal cost, and computational time for the three implementations were reported. The most optimal solutions, referred to as *admissible solutions*, were also further analyzed.

3 Results

The proposed workflow was successfully applied to the pendulum inversion problem without enlarging the computational time. The use of the continuity and spheres' penetration as penalties for the generation of the wise initial guess slightly increased the convergence rate to respectively 95.1% and 98.9% compared to 93.0% for the ***Constrained*** implementation. The use of our proposed workflow generated three techniques that were more optimal than the best solution found with the ***Constrained*** implementation (solutions to the left of the gray line in Fig. 1).

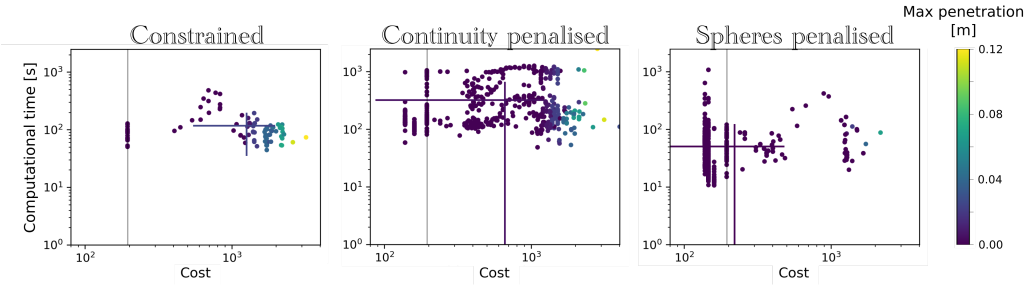

Figure 1: The optimal cost value, computational time and maximal spheres' penetration between the shooting nodes of the optimal solutions found for the constrained problem (left), the continuity included as a penalty (center) and the spheres' penetration included as penalties (right). All solutions shown in the figure satisfied the optimality criteria.

We considered the first four clusters of solutions to be *admissible solutions* (Fig. 2). The use of penalties also increased the rate of *admissible solutions* found to 43.7% and 89.4% for the ***Continuity penalised*** and ***Spheres penalised*** respectively compared to 19.4% for the ***Constrained*** implementation.

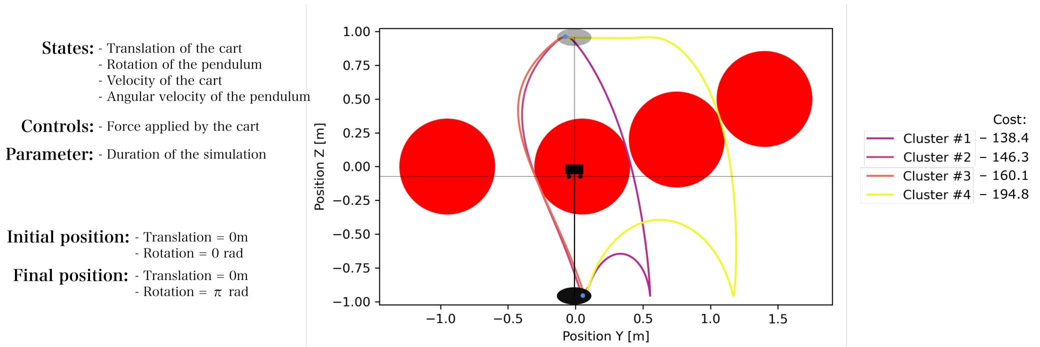

Figure 2: The pendulum's trajectory for the four *admissible solutions*. At the beginning of the simulation, the pendulum is hanging below the cart (black), then the cart has to move sideways on the rail to reach a perfectly inverted position (gray). The marker on the pendulum (blue dot) cannot get through the constraint spheres (red).

4 Conclusion

We proposed a two-step optimization workflow where constraints were transformed into penalties in the first step. The use of this workflow generated solutions that were more optimal and generated *admissible solutions* more often. The use of the proposed method is recommended for OCPs where multiple solutions are acceptable and it is not obvious beforehand which one should be prioritized.

References

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