Bond-graph representation of ideal constraints via the principle of virtual power

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EXTENDED ABSTRACT

1 Introduction

In a recent conference paper [1], the authors reported (but did not prove) that for a general system of constrained particles, having possible velocities represented by a partial velocity matrix, the partial velocity matrix is the modulus of a multibond-graph [2] modulated transformer element, which represents the subsystem of embedded constraints described by the partial velocity matrix, in the sense that the effort outputs of the transformer element are the particle-level constraint forces[3] for the embedded constraints. Although it is not the primary contribution of the paper, this fact is previously unreported in the literature.

2 Contribution

In this paper we provide a proof of the above-described fact, using the principle of virtual power. We further extend the result by deriving, from the same principle, the bond-graph representation of an additional set of adjoined constraints, which could be in general rheonomic, nonholonomic, and nonlinear in velocity [4]. This result is also unreported in the literature. To support the proofs, we provide a statement of the principle of virtual power which is well adapted to the classification of constraints as being either adjoined or embedded.

By combining the so-derived constraint subsystem bond graphs with an additional multibond subsystem, representing the particle kinetic energies, we develop a complete system-level bond graph (see Fig. 1), from which Kane's equations for the system can be easily derived. Such a bond-graphic derivation of Kane's equations, for a general particle system incorporating both embedded constraints and adjoined nonlinear nonholonomic constraints, has not been previously reported in the literature.

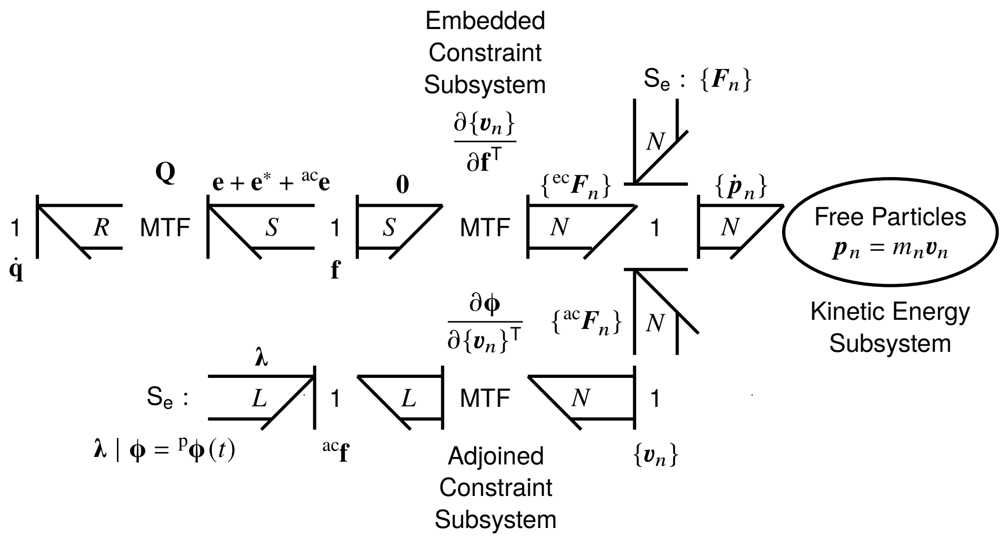

Figure 1: Bond-graph representation of constrained particle system

3 Discussion

By developing the system-level bond graph for a general system of constrained particles, we have found a result which could be further generalized to systems incorporating rigid or flexible bodies, by approximating such continuous bodies by a large but finite number of particles. The structure of the bond graph facilitates evaluation of the energy cost of rheonomic and nonlinear nonholonomic constraints, which are in general active constraints delivering power to the system.

[3] Both this paper and the previous paper make use of an extension to Breedveld's original multibond-graph dialect, allowing basis-free vectors as power-conjugate efforts and flows on multibonds [3].

The literature rarely reports the use of the principle of virtual power with bond graphs; but in this work we find that this variational principle is highly compatible with bond-graph representations. We further observe that the principle of virtual power is the lowest order variational principle of mechanics which can provide solutions for systems with nonlinear nonholonomic constraints [5], which certainly enhances its value in applications.

4 Conclusions

Although originally developed without reference to any variational principle [6, 7, 8], it has been recognized for some time that Kane's equations can be readily derived from the principle of virtual power [9, 10]. This paper strengthens that conclusion by providing a particle-level bond-graphic derivation of Kane's equations for a nonlinear nonholonomic system, the underlying bond graph having been developed by application of the virtual power principle separately to embedded and adjoined constraint subsystems.

We also call into question a customary practice in mechanics, which is to allow rheonomic constraints to be embedded in the initial system description. This practice is fundamentally incompatible with the bond-graph methodology, because it makes it impossible to determine the physical origins of time-based changes to the system kinetic energy. Our approach, which is no less general, is to defer the introduction of rheonomic constraints until the system has been completely described in terms of its generalized coordinates.

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