Configurational Forces and ALE Formulation for Geometrically Exact, Sliding Beams and Shells in Non-Material Domains

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EXTENDED ABSTRACT

1 Introduction

This paper addresses the problem of sliding beams shells, with special emphasis on situations where the sliding motion of the structure is not prescribed a priori. Hamilton's variational principle is used to derive the weak and strong forms of governing equations based on the systematic use of Reynolds' transport theorem. The strong form of the governing equations involve the mechanical and configurational momentum equations, together with the proper boundary conditions. It is shown that the configurational momentum equations are linear combinations of their mechanical counterparts and hence, are redundant. A weak form of the same equations is also developed; the configurational and mechanical momentum equations become independent because they combine in an integral form the strong mechanical and configurational momentum equations with their respective natural boundary conditions. The arbitrary Lagrangian-Eulerian (ALE) formulation is proposed and numerical examples are presented to contrast their relative performances [1, 2]. The predictions of both formulations are found to be in good agreement with those obtained from an ABAQUS model using contact pairs. Clearly, the proper treatment of the configurational forces impacts the accuracy of the model significantly.

2 Numerical Examples

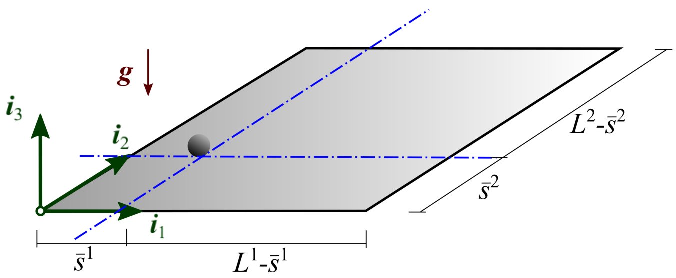

Figure 1: A moving mass on a simply supported shell.

Figure 1 depicts a point mass of 40 kg moving on a simply supported plate of thickness $h = 6$ mm, length $L^1 = 1$ m, and width $L^2 = 1$ m. The plate is made of steel with Young's modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$. At the initial time, the mass is located at the point ($\bar{s}^1 = 0.2$, $\bar{s}^2 = 0.3$) m on the plate, and constrained to slide. The mass-plate system deforms under gravity $g = 9.8$ m s^{-2} acting in the opposite direction of unit vector $\boldsymbol{i}_3$. At time $t = 0$, the mass is released and starts to slide on the plate. The plate also vibrates vertically due to the effects of the moving mass. Closed form solutions exists for this problem with the modal expansion technique.

The time history of position vector of the moving mass are shown in fig. 2 for in-plane components, r_1 and r_2, and in fig. 3 for out-of-plane component r_3, respectively. The out-of-plane displacement of the center of the plate is also shown in fig. 3. As expected, the concentrated mass moves back and forth in the range of $s^1 \in [\bar{s}^1,\ L_1 - \bar{s}^1]$ and $s^2 \in [\bar{s}^2,\ L_2 - \bar{s}^2]$. The present predictions agree well with closed-form solutions.

References

[1] S. L. Han. Configurational forces and geometrically exact formulation of sliding beams in non–material domains. *Computer Methods in Applied Mechanics and Engineering*, 395:115063, 2022.

[2] S. L. Han and O. A. Bauchau. Configurational forces in variable–length beams for flexible multibody dynamics. *Multibody System Dynamics*, 2023. To appear.

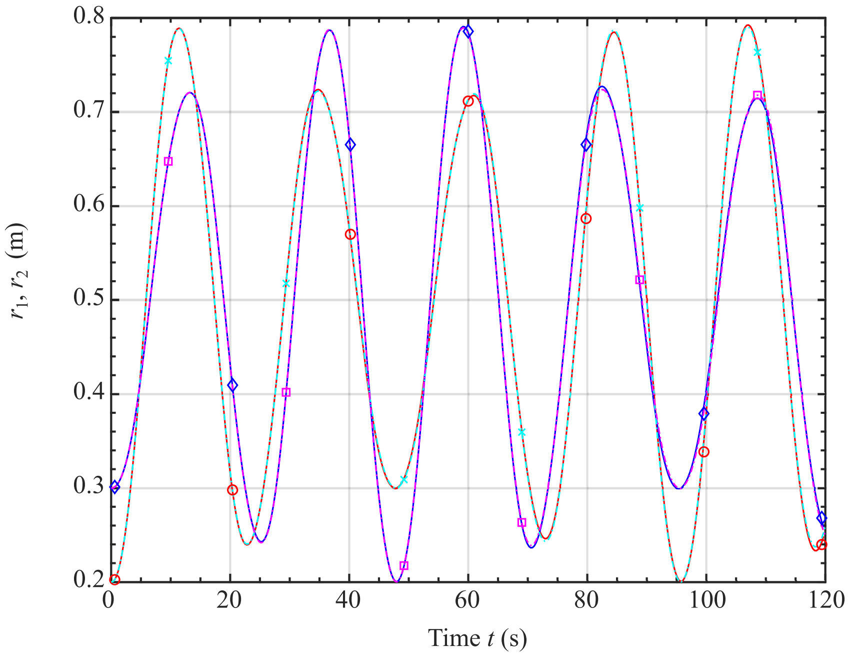

Figure 2: In-plane components of the position vector of the moving mass: present r_1 (○), present r_2 (◇), analytical r_1 (×), analytical r_2 (□).

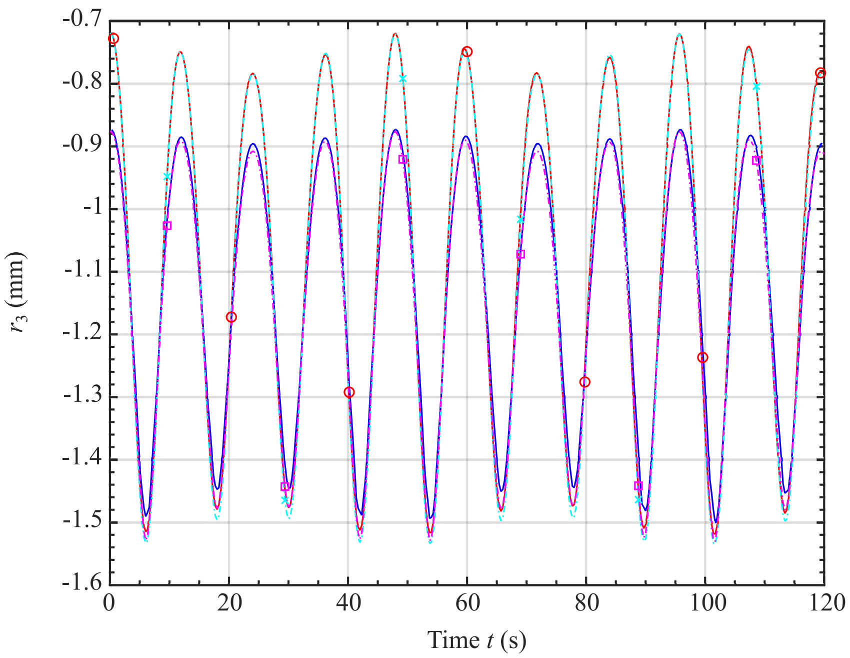

Figure 3: Out-of-plane component of the position vector of the moving mass and center of the plate: present for moving mass (○), analytical for moving mass (◇), present for plate center (×), analytical for plate center (□).