A general co-simulation framework based on a novel weak formulation at the interface level

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EXTENDED ABSTRACT

1 Introduction

Co-simulation or solver coupling has already been extensively utilized in various engineering fields [1]. The core idea consists in a decomposition of the global mechanical model into two (or more) submodels. The different subsystems are connected by coupling variables, which are exchanged only at discrete time-points, called macro-time (or communication) points. Among these points, the dynamics of each subsystem is determined independently, using its own solver. Generally, the subsystems are coupled by physical force/torque laws (applied forces/torques) or by algebraic constraint equations (reaction forces/torques) [2]. Furthermore, co-simulation approaches are subdivided into explicit, implicit and semi-implicit methods. Finally, regarding the decomposition of the global model into submodels, three possibilities are distinguished. Namely, force/force, force/displacement and displacement/displacement decomposition, in accordance with previous developments on the subject [2, 3].

Within this work, the focus is initially placed on a new scheme for the numerical integration of each subsystem, since the corresponding accuracy directly affects the correct and accurate solution of a decomposed model. Following that, the new co-simulation techniques are introduced. Specifically, a novel coupling strategy for satisfying the coupling conditions in their integral (weak) form, in the time domain, is proposed. This formulation constitutes a universal framework for the generation of coupling condition schemes with varying accuracy and stability properties, based on the choice of basis and polynomial order of the involved quantities. Then, the developed methods are initially applied to a linear oscillator model and to nonlinear pendulum models in order to ensure the validity of the proposed schemes. Finally, a full-scale vehicle-bridge interaction (VBI) problem is examined to demonstrate the applicability and effectiveness of the new methods in complex engineering applications.

2 Numerical integration

Initially, a new scheme for the numerical integration is presented based on previous work of the authors [4, 5]. Such methods are employed during the introduction of the new co-simulation methods for the integration of each subsystem. Specifically, through an appropriate three-field weak formulation [5], the equations of motion (EOMs) and the constraint equations are expressed as a convenient and consistent system of first order ordinary differential equations (ODEs), which carries over all the advantages of the corresponding second order ODE form [4]. The core difference lies in the choice of basis. Namely, the Haar wavelets (Fig. 1) are used in this work instead of piecewise-constant functions [5]. This allows the use of higher-order polynomials for the involved quantities and, thus, results in numerical integration schemes of improved accuracy and higher-order convergence rate.

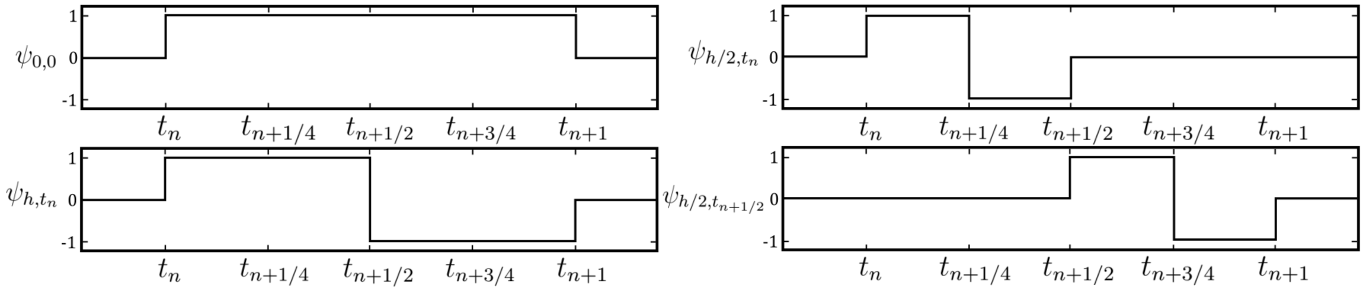

Figure 1: First four Haar wavelets

3 Co-simulation

In the present work, a general co-simulation framework for the generation of coupling condition schemes with preselected accuracy and stability properties is adopted. It is based on a novel weak formulation at the interface level and through a proper selection of basis (test functions) and polynomial order for the involved quantities. This implies that a new coupling strategy is introduced, which satisfies the coupling conditions over the time interval H in an average sense rather than strictly enforcing them only at discrete time points (the macro-time points). More specifically, the following conditions are imposed

$$\int_{T_N}^{T_{N+1}} W_1(f_1 - f_2)dt = 0 \tag{1}$$

$$\int_{T_N}^{T_{N+1}} W_2\left[(\overline{m}_{RR}\dot{\varphi})^{\cdot} + \overline{c}_{RR}\dot{\varphi} + \overline{k}_{RR}\varphi\right]dt = 0 \tag{2}$$

Based on these conditions, two different coupling schemes are then developed by employing piecewise-constant functions or Haar wavelets for the definition of the test functions. Clearly, the classical point-collocation method is a degenerate case of this general formulation by selecting the test functions as Dirac delta functions with non-zero values at the macro-time points only.

4 Numerical results and discussion

The new techniques are initially applied to a linear oscillator model with two masses, constrained with a fixed joint, as shown in Figure 2 (a). Subsequently, nonlinear models of a single and a double pendulum are examined for the new numerical integration and co-simulation methods, respectively. The main emphasis is placed on verifying the accuracy of the proposed schemes. In particular, a detailed analysis of the convergence and numerical error behaviour is carried out in the above-mentioned models. Typical numerical results are illustrated in Figure 2 (b), (c). Even though these models are relatively simple, the extracted results demonstrate the validity of the new numerical integration and co-simulation techniques.

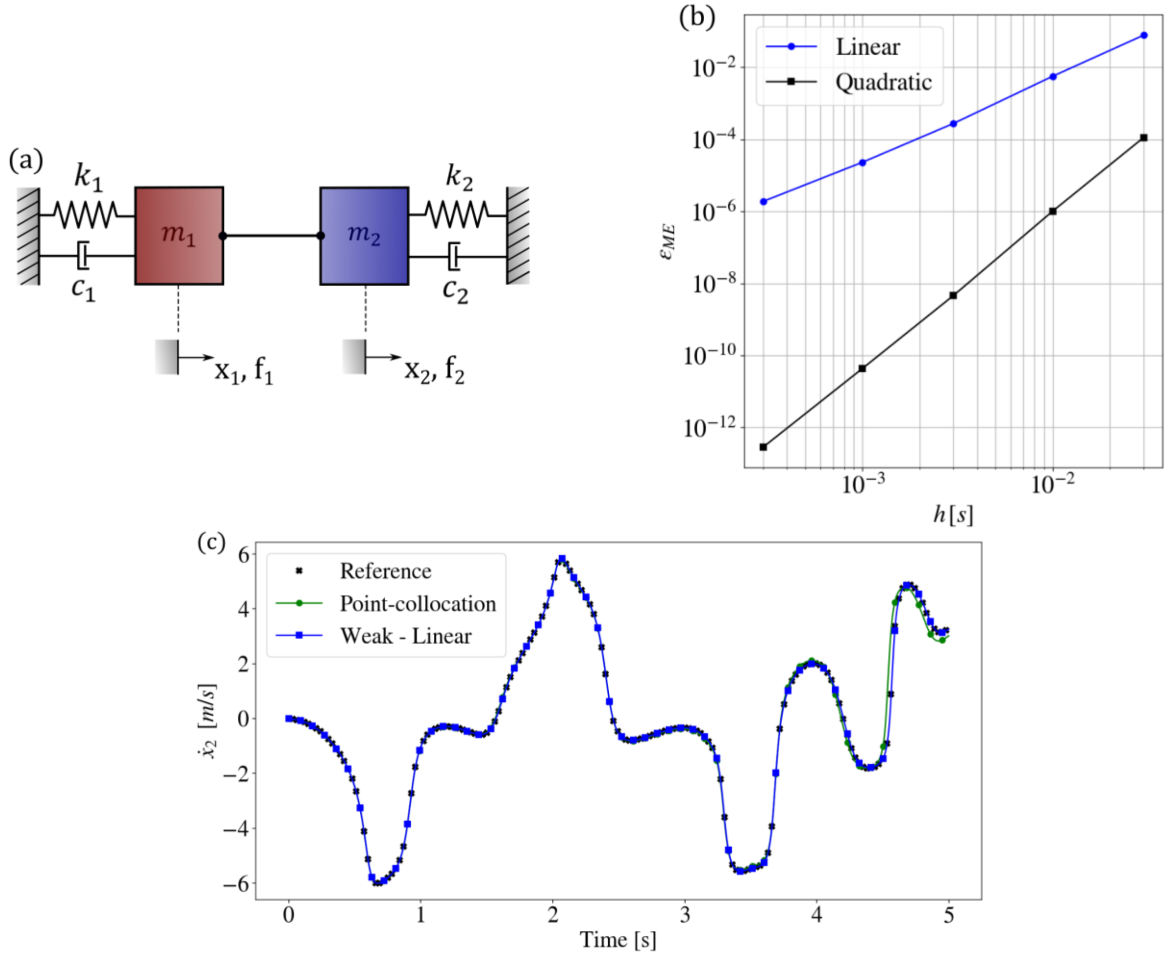

Figure 2: (a) A linear oscillator model and (b) typical numerical results for a nonlinear pendulum model and (c) for a nonlinear double pendulum model

Finally, the new techniques are extended and applied to a full-scale vehicle-bridge interaction problem. For this, the train structure is modeled as a multibody system, while the open-source finite element code OpenSeesPy is utilized in modeling the bridge structure. The extracted numerical results illustrate the applicability and effectiveness of the new methods in arbitrary and complex engineering problems.

References

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