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Modeling Viscoelastic Behavior in Flexible Multibody Systems

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EXTENDED ABSTRACT

1 Introduction

Viscoelasticity plays an important role in the dynamic response of flexible multibody systems. First, single degree-of-freedom joints, such as revolute and prismatic joints, are often equipped with elastomeric components that require complex models to capture their nonlinear behavior under the expected large relative motions found at these joints. Second, flexible joints, often called force or bushing elements, present similar challenges and involve up to six degrees of freedom. Finally, flexible components such as beams, plates, and shells also exhibit viscoelastic behavior. This paper presents a number of viscoelastic models that are suitable for these three types of applications. For single degree-of-freedom joints, models that capture their nonlinear, frequency-dependent, and frequency-independent behavior are necessary. The generalized Maxwell model is a classical model of linear viscoelasticity that can be extended easily to flexible joints. This paper also shows how existing viscoelastic models can be applied to geometrically exact beams, based on a three-dimensional representation of the quasi-static strain field in those structures. The paper presents a number of numerical examples for three types of applications. The shortcomings of the Kelvin-Voigt model, which is often used for flexible multibody systems, are underlined.

Flexible multibody systems are characterized by two distinguishing features: system components undergo finite relative motion and these components are connected by mechanical joints that impose constraints on their relative motion. Most studies focus on modeling of the dynamic response of the elastic components that undergo large motion and on the enforcement of the kinematic constraints at the joints. The analysis of the viscoelastic behavior of the structural components and that of the elastomeric devices often housed in the mechanical joints has received far less attention despite the fact that viscoelastic phenomena affect the dynamic response significantly.

In the context of flexible multibody systems, the tools used to analyze viscoelastic components fall into three categories, depending on the number of degrees of freedom used to describe their kinematics. In the first category, the kinematics of the problem is described by a single generalized coordinate. Two practical examples come to mind: a single degree of freedom, the relative rotation or displacement component, is used to describe the kinematics of elastomeric devices housed in revolute or prismatic joints, respectively. For revolute joints, the relationship between the moment and the relative rotation at the joint takes the form of a differential equation in time that could be nonlinear and could involve any number of additional state variables. For prismatic joints, the corresponding model expresses the force in the joint as a function of the relative displacement.

The basic concepts of classical viscoelasticity are often introduced based on one-dimensional rheological elements such as springs and dashpots [1, 2]. Typically, these models involve serial or parallel combinations of linear or nonlinear springs and dashpots to form increasingly complex models. For instance, the generalized Maxwell model consists of an elastic branch in parallel with any number of Maxwell fluid branches. The Kelvin-Voigt, Maxwell fluid, and Zener (also called the linear solid) models are particular cases of the generalized Maxwell model. Other authors [3] prefer a more abstract presentation based on differential equations in time of varying order. It is easy to prove the equivalence of many of these modeling approaches, which can often be presented either as differential equations or as convolution integrals.

In the second category, the kinematics of the problem is described by a few generalized coordinates. The flexible joint is a common element found in many multibody systems: it consists of a viscoelastic medium connected to two nodes of the model. The relative motion between these two nodes provides six generalized coordinates to describe the deformation of the medium, which generates three force and three moment components at the nodes. The basic viscoelastic models introduced in the previous paragraph can be generalized to deal with this more complex problem. Typically, the responses of the six generalized coordinates are coupled through both elastic and viscous phenomena that are important to capture.

For the last category, the attention turns to structural components such as beams, plates, and shells. The kinematic description of these flexible Cosserat solids requires both displacement and rotation fields. For instance, a beam is defined as a structure having one of its dimensions much larger than the other two. The generally curved axis of the beam is defined along that longer dimension and the cross-section slides along this axis. The cross-section's geometric and viscoelastic properties are assumed to vary smoothly along the beam's span. In aerospace applications, the cross-section of the beam could be a complex built-up structures involving highly anisotropic materials. Furthermore, some cross-sections include layers of material presenting energy dissipation characteristics in an effort to increase structural damping and control vibration.

2 Conclusions

Viscoelasticity plays an important role in the dynamic response of flexible multibody systems. For single degree-of-freedom joints, a number of nonlinear models were proposed: they capture the amplitude dependent, frequency-dependent, and frequency-independent behavior of elastomeric materials that are often used in revolute and prismatic joints.

For flexible joints, it was shown that the generalized Maxwell model can be extended easily to handle this six degree-of-freedom problem. The generalized Maxwell model is a classical model of linear viscoelasticity. If nonlinear behavior is observed in the joint, the nonlinear models presented for single degree-of-freedom joints can be used for each of the degrees of freedom independently.

Finally, a process was presented that allows the development of viscoelastic models for geometrically exact beams based on a three-dimensional representation of the quasi-static strain field in those structures. Two assumptions are at the heart of this process: the beam is undergoing low-frequency vibration and is lightly damped. The first assumption is inherent to beam theory; the second is an additional requirement for viscoelastic beams that is satisfied for commonly used structural materials.

A general approach to the problem was presented: starting from a three-dimensional viscoelastic material model, the corresponding viscoelastic beam model is constructed. The convolution integral that characterizes the generalized Maxwell model is found at the level of one-dimensional components, is generalized to three-dimensional materials, and is extended to cross-sectional models for beams. Although the presentation was based on the generalized Maxwell model, other viscoelastic models could be used. A similar process can be used to develop viscoelastic plate and shell models.

A number of numerical examples were presented. The viscoelastic model for geometrically exact beams was validated by comparing its predictions with those of a three-dimensional finite element code. Excellent agreement was observed for both displacement and local stress fields. Because it formulates a beam model, the computational efficiency of the proposed approach is several orders of magnitude higher than that achieved by three-dimensional finite element code.

As demonstrated in the numerical examples, the proposed approach is able to handle nearly-incompressible materials without exhibiting locking. This is particularly important because many materials used for their high energy dissipation characteristics, such as rubber and elastomeric materials, are nearly-incompressible.

Because it is based on the detailed knowledge of the strain field over the cross-section, the proposed approach is ideally suited to the analysis of beams of heterogeneous construction, where conventional structural materials and materials presenting enhanced energy dissipation characteristics are combined to achieve optimal performance. Local elastic and viscous stresses can be recovered easily at any point of the cross-section.

The use of the Kelvin-Voigt model is common practice to represent energy dissipation in flexible multibody systems. It was shown that this practice cannot capture the viscoelastic behavior of these systems accurately, and hence, should be avoided altogether.

The proposed approach can be generalized to viscoelastic materials featuring nonlinear material behavior; nonlinear behavior is common for elastomeric materials, for instance, even at low strain levels. In such case, the sectional analysis must be fully integrated with the solution of the beam equations, repeating the two-dimensional analysis at each time step during the simulation.

References

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